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Coping with the Collapse: A Stock-Flow Consistent Monetary Macrodynamics of Global Warming

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Coping with the Collapse: A Stock-Flow Consistent Monetary Macrodynamics of Global Warming. Updated version: January 5, 2017

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Summary

This paper presents a macroeconomic model of endogenous growth that takes into consideration the economic impact of climate change, the pivotal role of private debt and income distribution. Using a Goodwin-Keen approach, based on the Lotka-Volterra logic, we couple its nonlinear monetary dynamics of underemployment and income distribution with abatement costs. Various damage functions *à la* Nordhaus, Dietz-Stern, and Burke et al. reflect the loss in final production, stock of capital, and labor productivity due to the rise in temperature. An empirical calibration of our model at the world-scale enables us to simulate plausible trajectories for a planetary business-as-usual scenario. Our main finding is that, even though the short-run impact of climate change on economic fundamentals may seem *prima facie* rather minor, its long-run dynamic consequences may lead to an extreme downside. Under plausible circumstances, global warming forces the private sector to leverage in order to compensate for output and capital losses; the private debt overhang may eventually induce a global financial collapse, even before climate change could cause serious damage to the production sector. Under more severe conditions, the interplay between global warming and debt may lead to a secular stagnation followed by a collapse towards the end of this century. However, it turns out that increasing the wage share, fostering employment, or reducing the private-debt-to-output ratio makes it easier to avoid a collapse. The paper concludes by examining the conditions under which the +1.5°C and +2°C targets, adopted by the Paris Agreement (2015), could be reached thanks to an adequate carbon price trajectory.

Keywords : Ecological macroeconomics, Stock-Flow Consistent Model, Climate change, Integrated assessment, Collapse, Goodwin, Debt, Income distribution.

JEL Classification: C51, D72, E12, O13, Q51, Q54.

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Coping with Collapse: A Stock-Flow Consistent Monetary Macrodynamics of Global Warming*

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ABSTRACT

This paper presents a macroeconomic model of endogenous growth that takes into consideration the economic impact of climate change, the pivotal role of private debt and income distribution. Using a Goodwin-Keen approach ([25]), based on the Lotka-Volterra logic, we couple its nonlinear monetary dynamics of underemployment and income distribution with abatement costs. Various damage functions *à la* Nordhaus ([33]), Dietz-Stern ([10]), and Burke *et al.* ([5]) reflect the loss in final production, stock of capital, and labor productivity due to the rise in temperature. An empirical calibration of our model at the world-scale enables us to simulate plausible trajectories for a planetary business-as-usual scenario. Our main finding is that, even though the short-run impact of climate change on economic fundamentals may seem *prima facie* rather minor, its long-run dynamic consequences may lead to an extreme downside. Under plausible circumstances, global warming forces the private sector to leverage in order to compensate for output and capital losses; the private debt overhang may eventually induce a global financial collapse, even before climate change could cause serious damage to the production sector. Under more severe conditions, the interplay between global warming and debt may lead to a secular stagnation followed by a collapse towards the end of this century. However, it turns out that increasing the wage share, fostering employment, or reducing the private-debt-to-output ratio makes it easier to avoid a collapse. The paper concludes by examining the conditions under which the +1.5°C and +2°C targets, adopted by the Paris Agreement (2015), could be reached thanks to an adequate carbon price trajectory.

1 Introduction

Given the increasing awareness about climate change, which crystallized at a diplomatic level in the Paris Agreement of December 2015, and the growing concern about potential downside consequences of a temperature increase, the question is raised of whether global warming might *per se* induce a severe breakdown of the world economy. This paper tackles this issue and looks for policies designed to mitigate climate change through abatement costs. In particular, at Paris, nearly 200 countries promised to try to bring global emissions down from peak levels as soon as possible. More significantly, they pledged "to achieve a balance between anthropogenic emissions by sources and removals by sinks of greenhouse gases in the second half of this century." That means getting to net zero emissions between 2050 and the end of this century. The UN's climate science panel said net zero emissions must happen by 2070 to avoid dangerous warming —a claim reiterated at the COP22 summit of Marrakech (2016). Our results have implications on this

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question and provide a number of plausible conditions under which zero net emissions might even be required before 2050. Moreover, they shed new light on the feasibility of the +1.5°C and +2°C targets.

But beyond the physical issue of warming, the financial stake of the cost of mitigation and adaptation should not be neglected either. According to the New Climate Economy report (NCE, 2014[11]), US$ 90 trillion are needed at the world level over the next 15 years in order to fund clean infrastructures which will permit to reach zero net emissions; US$ 2 trillion per annum in high-income countries, and between US$ 3 and 4 trillion in low- and middle-income countries. These numbers prompt a daunting question: how will the world economy finance such monetary flows? Given today's vulnerability of public finances, it is expected that the private sector will be able to endorse the needed long-run investments. This, in turns, raises a new question: will the world economy be able to carry the corresponding additional private debt burden?[1]

As argued by Bank of England Governor Mark Carney, too rapid a movement towards a low-carbon economy could materially damage financial stability: "A wholesale re-assessment of prospects, as climate-related risks are re-evaluated, could destabilise markets, spark a pro-cyclical crystallisation of losses and lead to a persistent tightening of financial conditions: a climate Minsky moment" (Carney (2016)[6]). Conversely, insufficient take-up of adequate financial tools may prevent the world economy from investing at the required scale. Taking advantage of a growing body of literature in ecological macroeconomics,[2] we present an integrated ecological macroeconomic model that combines two sources of instability: (i) climate change and (ii) private over-indebtedness. By incorporating the latter into a rather low-dimensional stock-flow consistent economic model, we are able to track transmission channels between the two sources of potential economic breakdown alluded to by Carney. To the best of our knowledge, this paper offers the first macroeconomic stylized narrative where debt-deflation becomes the hallmark of a possible forthcoming breakdown caused by global warming. It confirms the view defended by Rezai, Taylor, and Mechler (2013)[36] that policy-relevant recommendations should be based on a holistic and macro perspective. Our hope is that it will pave the way towards the kind of (post-)Keynesian ecological macroeconomics advocated by these authors.

1.1 The climate and economy interaction

By contrast with the literature based on the Ramsey-Cass-Koopmans model, we incorporate endogenous drivers of growth and allow climate change to alter these channels. As argued by Stern (Stern (2006)[41]), climate change could have long-lasting impacts on growth. We borrow from the emerging body of empirical evidence pointing in this direction (e.g., Dell *et al.* (2012)[9]), even though climatic conditions in the recent past have been relatively stable compared to what we now have to contemplate for the near future.

Second, we consider various types of convexity of the damage function linking the increase in global mean temperature with the instantaneous reduction in output. That it might be highly convex at some temperature is strongly suggested by the literature on tipping points (see Dietz-Stern (2015)[10] and Weitzman (2012)[45]). By contrast, some existing integrated assessment model (IAM) studies assume very modest curvature of the damage function. The DICE default, for instance, is quadratic, and our simulations confirm that it leads to unrealistic narratives (see Section 4 below).

Third, we allow climate change to alter not only current output but also the stock of productive capital and labor productivity. Indeed, as argued by Dietz and Stern (2015)[10], various environmental phenomena such as the intensification of extreme events or sea level rise may directly damage capital and thus permanently dampen the level of output. Furthermore, the contributions of Burke and al. (2015)[5], Dafermos (2016)[8], or Dietz and Stern (2015)[10] highlight the fact that significantly high levels of global warming may affect the health of workers and their ability to perform tasks, thus diminishing their labor productivity.

1.2 The dynamics of debt

Since the financial crisis of 2007–2009, the ideas of Hyman Minsky concerning the intrinsic instability of a monetary market economy have experienced a significant revival. In this paper, we adopt a mathematical formalization of Minsky's standpoint in order to assess the role of debt dynamics in our narrative.[3] More precisely, our starting point is the basic Lotka-Volterra dynamics first introduced by Goodwin (1967)[19] and later extended by Keen (1995)[25]. Keen's model (1995)[25] is a three-dimensional non-linear dynamical system describing the time evolution of the wage share, employment rate, and private debt in a closed economy. Under reasonable assumptions, this system admits, among others, two locally stable long-run equilibria: one (the "good" equilibrium) with a finite level of debt and non-zero wages and employment rate, and a second (the "bad equilibrium") characterized by an infinite debt-to-output ratio, vanishing wages, and zero employment (Grasselli and Costa-Lima (2012)[22]). We show

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[1] See Giraud (2017)[14] for a first grasp of this issue.

[2] See, for instance, Dafermos (2016))[8] Dietz and Stern (2015)[10], or Nordhaus (2013)[35])

[3] Dos Santos (2005)[38] provides a survey up to 2005 of the literature on the modeling of Minskian instability; more recent contributions include Ryoo (2010)[37] and Chiarella and Guilmi (2011)[7].

how, absent any climatic complications, the world economy would converge towards a "good" steady state. The addition of a climate feedback, modeled through appropriately selected damage and abatement functions, may however drive the state of the economy towards the "bad" long-run equilibrium with unlimited debt, leading to a planetary downside.

1.3 Alternative modeling foundations

Over the past thirty years, many IAMs have been developed in order to estimate the impact of economic development on the environment. A solid body of literature compares IAMs, describing their advantages and disadvantages (Schwanitz (2013)[39]). The models considered in this literature fall into one of four categories based on the macroeconomic settings that they rely on: (1) welfare maximization; (2) general equilibrium; (3) partial equilibrium; and (4) cost minimization (Stanton *et al*. (2009)[40]). By contrast, our modeling approach assumes neither optimal behavior nor an equilibrium relation. It is worth mentioning that recent research has contributed to building alternatives to most IAMs by incorporating Keynesian features (see, e.g., Barker *et al.* (2012)[4]) or more post-Keynesian insights (see, e.g. Dafermos *et al.* (2017)[8]).

By way of illustration, the Ramsey-Cass-Koopmans model – the core economic model of DICE and the benchmark for IAM literature –, is a general equilibrium model of optimal savings that extends the Solow-Swan classical growth model. It represents a closed economy endowed with a constant return-to-scale Cobb-Douglas production function combining labor and capital. Capital goods are owned by households and hired at a market rental rate set endogenously. A perfectly competitive market is assumed, prices are measured in current units of output, and agents' decisions are made under perfect foresight. Outputs in the Ramsey-Cass-Koopmans model are driven by an optimal trajectory of the trade-off between consumption and investment that results from the maximization of an intertemporal utilitarian utility function aggregating households' preferences. As a result, households perfectly anticipate the optimal budget path, imposing a non-deviation from the optimal trajectory, which is precluded by an intertemporal budget constraint (transversality condition). At the steady state, the so-called modified golden rule of Keynes-Ramsey is verified – the net marginal product should equal the required rate of return taking into account the pure rate of time preference and the desire to smooth consumption. Finally, output increases at the pace of labor force growth and technological progress. It is worth mentioning that such a dynamic precludes situations such as mass unemployment and over-indebtedness.

At variance with such models, our modeling approach is based on the myopic behavior of imperfectly competitive firms, allows for multiple long-run equilibria, is stock-flow consistent (Godley and Lavoie (2012)[18]), and exhibits endogenous monetary cycles and growth, sticky prices, private debt, and underemployment. Moreover, money is endogenously created by the banking sector (Giraud and Grasselli (2016)[15]) and turns out to be non-neutral (Giraud and Kockerols (2016)[16]). The non-trivial properties of money enable the emergence of phenomena such as debt-deflation (Grasselli *et al*. (2015)[23]). Here, by contrast with general equilibrium approaches (see, e.g., Giraud and Pottier (2015)[17]), debt-deflation need not just appear as a "black swan" – or, more precisely, a "rare" event relegated to the tail of risk distribution. On the contrary, depending on the basin of attraction into which the state of the economy is driven by climate damages, the ultimate breakdown may occur as the inescapable consequence of the business-as-usual (BAU) trajectory.

The paper is organized as follows: Section 2 introduces the the stock-flow consistent macroeconomic model, its climate counterpart, and the interconnection channels through which the output level, emissions, CO_2 concentration, the average atmosphere temperature increase, and damages induced by climate change will be set. Section 3 provides some mathematical and numerical insights into the destabilizing impact of climate change on our modeling. Section 4 discusses the different scenarios arising from the interplay of our various key parameters and addresses the emblematic objective of $+2°$C and approaching as closely as possible the $+1.5°$C global warming target adopted by the Paris Agreement. The final section summarizes the main conclusions and outlines areas for future research.

2 An integrated framework

Our IAM depicts the interrelations between a global monetary economy and climate change. Even though, for simplicity, the public sector is not explicitly modeled, public policy objectives materialize through an emission reduction rate that can be achieved via the deployment of a carbon price instrument.[4]

The core macroeconomic module in the absence of climate change is presented in subsection 2.1 and the climate module in subsection 2.2. The introduction of damages and the way these are controlled through public policy objectives is discussed in subsection 2.3. The calibration of the parameters introduced throughout this section is provided in Appendix A.

2.1 Monetary macrodynamics

Our underlying macroeconomic model closely follows the contribution of Grasselli and Costa-Lima (2012)[22] and the literature centered around Keen's (1995)[25] approach,

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[4] Public intervention, as well as the role of public debt, will be analyzed in depth in a subsequent paper.

such as Graselli *et al.* (2012, 2014, 2015)[22],[21],[23] and Nguyen-Huu *et al.* (2014)[31] among others. This framework, based on a Lotka-Volterra logic, is motivated by the aftermath of the 2008 subprime mortgage crisis, during which private debt played a pivotal role in endangering the world's macroeconomic stability. One appeal of this literature lies in its ability to formalize economic collapse as a consequence of over-indebtedness.[5]

2.1.1 Production

Following the seminal work of Goodwin (1967)[19], we assume that firms produce a real amount, Y, of a unique consumption good combining labor and capital through a Leontief technology:

$$Y = \frac{K}{\nu} = aL. \tag{1}$$

K and L refer respectively to the stock of capital and labor, $1/\nu$ and a stand respectively for (constant) capital productivity and Harrod-neutral labor-augmenting progress. The capital-output ratio, ν, is assumed to be constant. For simplicity, as shown by Eq. 1, production factors are assumed to be always used at full capacity.

2.1.2 Firms

Firms myopically produce goods according to the current level of capital. The dynamic of the latter is shaped by the investment resulting here from the firms' decisions, in which the current nominal profit rate plays a prominent role, as described in Eqs. 2 – 9.

$$\Pi := pY - wL - rD, \tag{2}$$

$$\omega := \frac{wL}{pY} \text{ and } d := \frac{D}{pY}, \tag{3}$$

$$\pi := 1 - \omega - rd, \tag{4}$$

$$I := \kappa(\pi)Y, \tag{5}$$

$$K := I - \delta K, \tag{6}$$

$$\dot{D} := pI + D_i(\pi) - \Pi, \tag{7}$$

$$D_i := \Delta(\pi)pY, \tag{8}$$

$$i := \frac{\dot{p}}{p} := \eta_p(m\omega - 1) + c. \tag{9}$$

Denoting p as the price of the consumption good, the nominal net profit of firms, Π, is defined in Eq. 2 as the nominal output *minus* the wage bill and the private debt burden – where $r \geq 0$ is the short-term interest rate[6] and D, the outstanding balance of current nominal private debt. Defining both the nominal wage share, ω, of the economy and its private debt ratio, d, by Eq. 3, the nominal profit share, π, can now be defined in Eq. 4.

Following Keen (1995)[25], real investment, I, is driven by the profit share, π, capturing the risk appetite of firms. The increasing real-valued function, $\kappa(\cdot)$, introduced in Eq. 5, will be empirically estimated.[7] Capital obeys the standard rule of accumulation expressed in real terms in Eq. 6, where $\delta > 0$ stands for the constant rate of capital depreciation.

Eq. 7 models the evolution of the nominal private corporate debt, D, depending on the gap between current nominal profit, Π, and nominal investment, pI, *plus* nominal dividends paid to the firms' shareholders, D_i. According to Eq. 8, the current level of nominal dividends, viewed as a fraction of nominal output,[8] is an increasing real-valued function, $\Delta(\cdot)$, of the profit rate, π.[9] Moreover, the profits from the banking sector, rD, are redistributed to the shareholders. Thus, the whole income that accrues to households is $W + Di + rD$.

Finally, Eq. (9) captures the dynamics of inflation, where the long-run equilibrium price is given by a markup, $m \geq 1$, times the unit labor cost, $W/pY = w/pa$. Absent any price hysteresis, the latter would converge to this long-run target through a lagged adjustment of exponential form with a relaxation time, $1/\eta_p$. Whenever the consumption goods market is imperfectly competitive, $m > 1$.[10] In addition, we assume that current prices depend on their historical path inasmuch as they follow a trend, c, driven by the mean of past inflation. This dynamics of inflation will be empirically calibrated.

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[5] See Giraud and Grasselli (2016)[15] for an explicit modeling of the dynamics of households' debt.

[6] Here, for simplicity, r is kept constant.

[7] See the supplementary web material for details on the estimation process of $\kappa(\cdot)$, as well as on function $\phi(\cdot)$ and Eq. 9, to be defined shortly. We refrain from providing micro-foundations to either $\kappa(\cdot)$ or $\phi(\cdot)$ (to be introduced in Eq. 12). Indeed, as shown by Mas-Colell (1995)[29], when full-blown rational corporates are sufficiently numerous and heterogeneous, they are exposed to an "everything-is-possible" theorem *à la* Sonnenschein-Mantel-Debreu at the aggregate level. Our phenomenological approach takes due account of this emergence phenomenon.

[8] One can argue that it would make more sense to view Di as a fraction of Π. At the time of this writing, however, some major oil companies are financing the dividends they pay to their shareholders through additional leverage – which confirms our specification.

[9] The reader should note that behavioral functions have been bounded to avoid inconsistent behaviour that might fall far outside the estimation range.

[10] The parameter η_p plays a role analogous to the Calvo parameter in the neo-Keynesian literature.

2.1.3 The labor market

The global workforce, N, is assumed to grow according to a sigmoid inferred from the 15–64 age group in the United Nations scenario[1]:

$$\beta := \frac{\dot{N}}{N} = q\big(1 - \frac{N}{P^N}\big). \tag{10}$$

$P^N \approx 7.056$ billion stands for the upper bound of the world's labor force and q for the speed of convergence towards P^N.[11] The employment rate is defined in Eq. 11 as the ratio between the number of employed workers, L, and the global labor force, M:

$$\lambda := \frac{L}{N}. \tag{11}$$

The link between the real and nominal spheres of the economy is provided by a short-run wage-price dynamics taken from Grasselli and Nguyen-Huu (2014)[24].[12]

$$\frac{\dot{w}}{w} := \phi(\lambda) + \gamma i. \tag{12}$$

In other words, workers bargain for their wages, w, based on the current state of employment, λ, (as in Keen (1995)[25]) through some increasing real-valued function, $\phi(\cdot)$, which will be empirically estimated. They also take into account the observed inflation rate, i, resulting from firms' price-setting. The constant $\gamma \in [0;1]$ measures the degree of monetary illusion, with $\gamma = 1$ corresponding to the case where workers fully incorporate inflation into their bargaining (no "money illusion").

Finally, as in both the Goodwin and Keen models, the behavior of households is postulated to be fully accommodating in the sense that, given investment, consumption is determined by the well-known macro balance:

$$C := Y - I. \tag{13}$$

2.2 The climate module

Our formalization of emissions and climate change closely follows the conventional framework of integrated assessment models, and in particular the DICE model introduced by Nordhaus in his seminal work (1993, 2014)[32] [34], adapted here to our continuous time framework.[13]

$$E := E_{ind} + E_{land}, \tag{14}$$

$$E_{ind} := Y\sigma(1-n), \tag{15}$$

$$\frac{\dot{\sigma}}{\sigma} := g_\sigma, \tag{16}$$

$$\frac{\dot{g_\sigma}}{g_\sigma} := \delta_{g_\sigma}, \tag{17}$$

$$\frac{\dot{E}_{land}}{E_{land}} := \delta_{E_{land}}, \tag{18}$$

$$\dot{\mathrm{CO}_2}^{AT} := E - \Phi_{12}\left(\mathrm{CO}_2^{AT} - \frac{C_{AT_{pind}}}{C_{UP_{pind}}}\mathrm{CO}_2^{UP}\right) \tag{19}$$

$$\begin{aligned}\dot{\mathrm{CO}_2}^{UP} &:= \Phi_{12}\left(\mathrm{CO}_2^{AT} - \frac{C_{AT_{pind}}}{C_{UP_{pind}}}\mathrm{CO}_2^{UP}\right) + \ldots \\ &\quad \ldots - \Phi_{23}\left(\mathrm{CO}_2^{UP} - \frac{C_{UP_{pind}}}{C_{LO_{pind}}}\mathrm{CO}_2^{LO}\right),\end{aligned} \tag{20}$$

$$\dot{\mathrm{CO}_2}^{LO} := \Phi_{23}\left(\mathrm{CO}_2^{UP} - \frac{C_{UP_{pind}}}{C_{LO_{pind}}}\mathrm{CO}_2^{LO}\right), \tag{21}$$

$$F := F_{ind} + F_{exo}, \tag{22}$$

$$F_{ind} := \frac{F_{2\times CO_2}}{\log(2)}\log\left(\frac{C_{CO_2}}{C_{AT_{pind}}}\right), \tag{23}$$

$$\dot{F}_{exo} := \delta_{F_{exo}}F_{exo}\left(1 - \frac{F_{exo}}{F_{exo}^P}\right), \tag{24}$$

$$C\dot{T} := F - \rho T - \gamma^*(T - T_0), \tag{25}$$

$$C_0\dot{T}_0 := \gamma^*(T - T_0). \tag{26}$$

As shown by Eqs. 14–18, global CO_2 emissions are the sum of two terms: (i) industrial emissions, E_{ind}, linked to real output and (ii) land-use emissions, E_{land}.[14] The latter source of emissions is exogenous and meant to decrease at the rate $\delta_{E_{land}} < 0$. The level of industrial emissions defined in Eq. 15 depends on the current emission intensity of the economy, σ,[15] the mitigation efforts through the emission-reduction rate, n, defined shortly, and the output level of the economy.

The carbon cycle is represented in Eqs. 19–21 through an interacting three-layer model figuring: (i) the atmosphere (AT); (ii) a mixing reservoir in the upper ocean and the biosphere (UP); and (iii) the deep ocean (LO), in which

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[11] The details of the calibration of these parameters are given in the supplementary web material.

[12] See also Mankiw (2010)[28].

[13] DICE uses a two-layer model as in Geoffroy *et al.* (2013)[13]. The only difference between the two models lies in the fact that the first is built in discrete time while the second runs in continuous time. However, we used the trajectories of the DICE model to calibrate our continuous time version.

[14] In concrete terms, this second contribution can be viewed as being induced by deforestation and the implied release of CO_2.

[15] The dynamics of σ is given by Eqs. 16 and 17, where $\delta_{g_\sigma} < 0$ is a parameter controlling the exogenous decrease of emission intensity.

global CO_2 emissions, E, accumulate. When the level of emissions is null following completion of the energy shift, the total amount of CO_2 (existing and released) will spread according to the diffusion parameters $\Phi_{ij}, i \in \{1,2,3\}$, such that the relative pre-industrial concentrations $C_{i_{pind}}, (i,j) \in \{AT, UP, LO\}^2$ in each layer are respected at equilibrium.

The accumulation of greenhouse gases modifies the chemical properties and thus the energy balance of the atmosphere layer, triggering a rise in the so-called radiative forcing F of CO_2 as modeled by Eqs. 22–24. A distinction is made between industrial forcing F_{ind} (from CO_2) and residual forcing,[16] F_{exo} (resulting from various residual factors such as non-CO_2 long-lived greenhouse gases and other factors such as albedo changes, or the cloud effect). One can note that, in Eq. 23, the parameter $F_{2\times CO_2}$ represents the increase in the radiative forcing resulting from a doubling of the pre-industrial CO_2 concentration.

Finally, the rise in radiative forcing induces a change, T, in the global mean atmospheric temperature as modeled in Eqs. 25–26. The global thermal behavior results from a coupled two-layer energy-balance model that roughly represents: (i) the atmosphere, land surface and upper ocean with a mean temperature, T, and (ii) the deeper ocean with a mean temperature, T_0. In this framework, the latter layer represents the long-run thermal inertia effects of the climate system. The remaining parameters are: ρ, the radiative feedback parameter; γ^*, the heat exchange coefficient between the two layers; C, the heat capacity of the atmosphere, land surface and upper ocean layer; and C_0, the heat capacity of the deep ocean layer. As Geoffroy *et al*. (2013)[13] point out, this formalism makes it possible to account for the two-frequency responses of the mean atmospheric temperature change through a distinct Transient Climate Response (TCR) and an Equilibrium Climate Sensitivity (ECS, determined by $T = F/\rho$ in this framework, which would allow us to perform some sensitivity analyses, as Dietz-Stern (2015)[10] have done).[17]

2.3 Damages and mitigation

The macroeconomic and climate modules are coupled through: (i) an environmental damage function that quantifies the real economic loss due to global warming and (ii) mitigation efforts implemented through the deployment of a carbon price instrument. Finally, achieving the energy shift will impose a carbon abatement cost on the economy.

2.3.1 Environmental damages

The damage function summarizes the economic impacts brought on by the rise in mean atmospheric temperature. It thus has to compile a wide range of events, including biodiversity loss, ocean acidification, sea level rise, change in ocean circulation, and highly frequent storms, among others, and consequently exhibits highly nonlinear and threshold effects. Conventional damage functions, as introduced by Nordhaus in his seminal work (2013)[35], are designed to express the aggregate economic impact of climate change as a fraction of current real output. However, as pointed out by Dietz and Stern (2015)[10] and Dafermos *et al*. (2016)[8], global warming may have an adverse impact not only on throughput but also on the factors of production themselves, as well as their levels of productivity. We thus introduce three models of damage for the purpose of this paper.

a) Damage to output only

For our first specification, we consider environmental damages conventionally modeled as affecting output alone. We rely on the functional form and calibration provided by Dietz and Stern (2015)[10] with a polynomial damage function altering real production as described in Eqs. 27–28

$$\mathbf{D} := 1 - \frac{1}{1 + \pi_1 T + \pi_2 T^2 + \pi_3 T^{\zeta_3}}, \tag{27}$$

$$Y := (1 - \mathbf{D})\frac{K}{\nu} = (1 - \mathbf{D}) a L. \tag{28}$$

This formulation was initially introduced by Weitzman (2012)[45]. It assumes the same calibration as Nordhaus (2013)[35] for the quadratic part, while the parameters π_3 and ζ_3 are calibrated so that the tipping points of temperature deviation and welfare loss equivalents are met.[18] A variant of this formulation was suggested by Dietz and Stern (2015)[10] and will also be considered in this paper.

By way of illustration, Figure 1 plots the shapes of the different damage functions considered in this paper.

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[16] For simplicity, the residual forcing is taken here as exogenous, as shown by the IPCC (2013)[43] to be negligible and in line with representative concentration pathways.

[17] TCR and ECS represent the mean atmospheric temperature deviations, at different time scales, induced by the change of radiative forcing resulting from a linear doubling of the atmospheric CO_2 concentration (at rate of a 1% increase of the stock per annum, hence a doubling in about 70 years). The TCR denotes the deviation obtained at the end of this doubling, while the ECS accounts for the new equilibrium of the system, reached decades later due to its thermal inertia. In our modeling, and assuming the calibration given in Appendix A, we find a TCR of approximately 1.5, which is in line with the Fifth Assessment Report of the IPCC (2013)[43]

[18] More precisely, the calibration relies on educated guesses about the loss of output for given temperature thresholds at some of the climate system's tipping points.

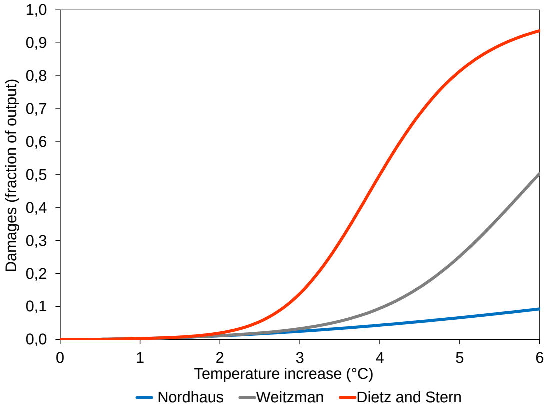

Figure 1: Shape of damage functions.

b) Damage to output *and* capital

Next, following the contributions of Dafermos (2016)[8], Dietz and Stern (2015)[10], and Moyer *et al.* (2014)[30], we allocate the impact of damages to output *and* capital according to Eqs. 29–32. In this specification, environmental damages alter not only output but also the stock of capital through various phenomena such as a rise in sea level, the intensification of extreme events, or the increase of anthropogenic pressure on land, due to soil erosion, for instance.

$$\mathbf{D}^{\mathrm{K}} := f_K\,\mathbf{D}, \tag{29}$$

$$\mathbf{D}^{\mathrm{Y}} := 1-\frac{1-\mathbf{D}}{1-\mathbf{D}^{\mathrm{K}}}, \tag{30}$$

$$Y := (1-\mathbf{D}^{\mathrm{Y}})\frac{K}{\nu} = (1-\mathbf{D}^{\mathrm{Y}})aL, \tag{31}$$

$$\dot{K} := I-(\delta+\mathbf{D}^{\mathrm{K}})K. \tag{32}$$

The total amount of damages defined by Eq. 27 is now distributed between the production of goods and the dynamics of capital, as described in Eqs. 31 and 32. The allocation rule given in Eqs. 29 and 30 ensures that the *instantaneous* level of damages will be identical in either specification a) or b).[19] However, the *dynamic* effects of this second specification will be more severe as the stock of capital, which drives the potential output of the economy, is now penalized.

c) Damage to labor productivity and capital

In our third and last specification, we adopt an alternative definition of the damage function introduced by Burke *et al.* (2015)[5]. This time, climate change directly alters the labor productivity according to:

$$\alpha := \frac{\dot{a}}{a} = \alpha_1 T_a + \alpha_2 T_a^2. \tag{33}$$

In Burke *et al.* (2015)[5], a comprehensive econometric model of the dependency of world GDP growth on climate parameters is provided.[20] In particular, a quadratic relationship between the mean annual temperature and income growth is introduced, from which we deduce 33, where T_a stands for the absolute atmospheric temperature, $T_a = T_{preind} + T$ with T_{preind} the pre-industrial temperature, and α_1, α_2 are estimated by Burke *et al.* (2015)[5].

As in the previous specification, we allow climate change to damage capital according to Eqs. 29 and 32. In fact, damages to capital will presumably be caused by catastrophes that have not yet happened, and are thus not captured by the historical data analyzed in Burke *et al.* (2015)[5]. However, in order to avoid any possible double accounting, output is no longer assumed to be reduced by global warming, so that 1 holds.

2.3.2 Mitigation efforts

While the passive decline of the energy intensity, σ, slowly improves the environmental performance of the economy, an active emission-reduction strategy is also implemented under the supervision of some public authority in order to control the pace of the energy shift, which requires that specific levels of the carbon emission-reduction rate, n, be met. This can be achieved through the deployment of various public policy instruments, which for simplicity, are modeled here through an exogenously given carbon price schedule, p_C:[21]

$$\frac{\dot{p}_C}{p_C} := \delta_{p_C} \geq 0. \tag{34}$$

A backstop technology is also available at price p_{BS}:

$$\frac{\dot{p}_{BS}}{p_{BS}} := \delta_{p_{BS}} \leq 0. \tag{35}$$

The emission reduction rate, n, then results from the arbitrage between this carbon price and the decreasing deployment cost of the backstop technology: once the carbon

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[19] More precisely, both specifications give the same total amount of damages. Indeed, if the total amount of damages defined by Eq. 27 is d% of the total output, then the second specification, defined by the sum of Eqs. 29–32, will also penalize the total output by the same amount d%.

[20] Burke *et al.* (2015)[5] implement a first-difference panel regression assessing a quadratic temperature impact on GDP growth with fixed effects on countries and periods, flexible country-specific trends, and precipitation controls (quadratic impact). Their methodology is robust and copes with both observed and unobserved effects such as nonlinear country-specific demographic trends. They propose a range of 17 models of regression studying several samples, an additional explanatory variable (developed and developing countries), and an alternative data source (the Penn World Tables, although the World Bank is their main source). It is worth mentioning that Burke *et al.* (2015)[5] provide estimates based only on the temperature effects provided by historical data.

[21] p_C refers to the price per ton of CO_2-e_2.

price equals the backstop technology price, the energy shift is automatically completed.[22]

$$n := \min\left\{\left(\frac{p_c}{p_{BS}}\right)^{\frac{1}{\theta_2-1}};1\right\}. \tag{36}$$

The parameter θ_2 controls the elasticity of substitution between the "clean" and the "dirty" technologies.

2.3.3 Carbon abatement costs

Carbon emissions abatement is achieved at some cost, GY, which can be borne either by investment or by consumption. This cost conveys the idea that some of the goods and services (investment or consumption) do not fulfil their original purpose but are instead used to decarbonize the economy.

The carbon emissions abatement cost, G, defined as a fraction of real output, Y, is proportional to the emission intensity, σ, and the price of the backstop technology, p_{BS}:

$$G := \theta_1 \sigma p_{BS} n^{\theta_2}. \tag{37}$$

In general, this cost is borne by investment at the rate $\mu \in [0;1]$, the remaining part, $(1-\mu)$, being borne by households. Here, for the sake of simplicity, we will restrict ourselves to $\mu = 1$, so that households are not impacted by abatement costs.[23] Firms continue to dedicate (part of) their profits to investment, as defined in Eq. 5, so that the dynamics of private debt described in Eq. 7 remains unchanged. However, the abatement cost μGY borne by firms is now deduced from this *gross* investment so that only the *effective* investment, I^{ef},

$$I^{ef} := (\kappa(\pi) - \mu G)Y. \tag{38}$$

is converted into gross formation of fixed capital according to the standard rule of capital accumulation:

$$\dot{K} := I^{ef} - \delta K. \tag{39}$$

Thus, a fraction of the funds invested by firms is used to decarbonize the current production apparatus and does not result in the acquisition of new machinery.

2.4 Wrap-up: stock-flow consistency

Table 1 displays the stock-flow consistency of our model. It can be readily checked that, in this set-up, the accounting identity "investment = saving" always holds.

	Households	Firms		Banks	Sum
Balance Sheet					
capital		pK			pK
Deposits	M^h	M^f		$-M$	
Loans		$-L$		L	
Equities	$E^b + E^f$	$-E^f$		$-E^b$	
Sum (net worth)	X^h	X^f		X^b	X
Transactions		current	capital		
Consumption	$-pC$	pC			
Investment		pI	$-pI$		
Accounting memo [GDP]		$[pY]$			
Wages	W	$-W$			
Dividends	$Di + r(L-M)$		$-Di$	$-r(L-M)$	
Interests on loans		$-rL$		rL	
Interests on deposits	$+rM^h$	$+rM^f$		$-rM$	
Financial Balances	S^h	Π	$-pI-Di$	0	
Flow of Funds					
Gross Fixed Capital Formation		pI			pI
Change in deposits	$\dot{M}^h$	$\dot{M}^f$		$-\dot{M}$	
Change in loans		$-\dot{L}$		$\dot{L}$	
Change in equities	$\dot{E}^f + \dot{E}^b$	$-\dot{E}^f$		$-\dot{E}^b$	
Column sum	S^h	$\Pi - Di$		0	pI
Change in net worth	$\dot{X}^h = S^h$	$\dot{X}^f = \Pi - Di + \left[\dot{p} - (\delta + \mathbf{D}^K + \frac{G}{\nu})p\right]K$		$\dot{X}^b = 0$	$\dot{X} = pI + \left[\dot{p} - (\delta + \mathbf{D}^K + \frac{G}{\nu})p\right]K$

Table 1: Balance sheet, transactions, and flow of funds in the economy

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[22] We plan to replace this simplistic narrative by a more realistic description of the energy shift in a subsequent work that will rely on a Putty-Clay approach.

[23] The more general situation will be dealt with in a companion paper.

Table 1 makes explicit the monetary counterpart of our economy: M stands for total deposits and equals M^h, t deposits of households, *plus* M^f, the deposits of firr $D := L - M^f$ represents the net borrowing of non-financ firms, i.e., loans, L, *minus* firms' deposits. Since divider of both financial and non-financial entities are redistribut to households, the latter own both the firms' and the ban equities, resp. E^f and E^b. Notice that, since the banks' nancial balance is always zero, their equity, E^b, can saf be assumed constant. Similarly, we assume that the marl value of the firms' equity is constant (because stock mark are closed in this model). Moreover, it follows from the counting equations $pY = \Pi + W + rD = pC + pI$ and Eq that $W + D_i + rD = \dot{D} + pC$, so that $\dot{M}^h = \dot{D} = \dot{L} - \dot{M}^f$. T change in households' net worth, $\dot{X}^h$, is thus the change private debt, $\dot{D}$.

3 Analysis of the long-term stea states

This section aims to study the long-term consequen of climate-induced damages on our macroeconomic fran work. In order to begin to understand how climate fects the macro-economic path of the world economy, need to disentangle the various mechanisms that inter with each others: emissions, damages, inflation, unempl ment, income distribution, debt... Let us therefore begin with the *Baseline scenario*, which is a business-as-usual trajectory *without* a climate feedback loop. This will provide a macroeconomic benchmark, absent climate considerations.

3.1 The Baseline case

Figure 8 below presents the trajectory obtained in the Baseline case and Table 3 some of its key figures. The (exogenous) deterministic exponential productivity growth drives the exponential growth of real GDP, which in 2100 reaches 11 times its initial 2010 volume. This uninterrupted growth is accompanied by endogenous monetary and real cycles with a periodicity of 12–18 years – thus close to the Kuznets business swings (cf. Kuznets [26]). On a large time scale, however, the magnitude of each cycle tends to decrease, and the state of the economy converges towards a long-run equilibrium: while output still grows exponentially, the endogenous volatility of most parameters tends to zero, and a phenomenon akin to a worldwide "Great Moderation" occurs. The employment rate oscillates around 72%, and the wage share converges in the vicinity of 63%. At variance, however, with the infamous Great Moderation observed in the decade preceding the global financial crisis of 2007–2009, here, the debt-to-output ratio stabilizes at

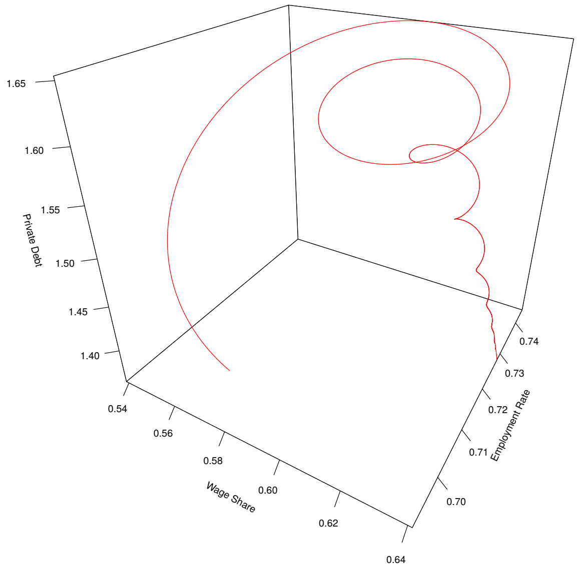

Figure 2: Phase diagram (ω, λ, d) for the Baseline scenario (period 2010–2300).

The underlying market forces shaping these endogenous cycles interact as follows. As output grows, more workers are employed, which eases labor negotiations and, courtesy of the short-run Phillips curve (Eq. 12), induces an increase of the wage share, ω. As a result, inflation tends to accelerate (Eq. 9).[25] As shown by Eq. 4, this process devours the profit share, π, hence reducing investment and credit (see Eqs. 7–5). The slowdown of capital growth then results in a lower output growth, thus reducing the employment rate. This reversal in trend cools down the wage growth rate, restoring the profit share and hindering cost-push inflation. Next, the higher profitability turns out to induce a sufficiently strong revival of investment, aggregate demand overcomes the initial relative reduction in consumption due to the redistribution of income from workers to investors, and output growth accelerates again. According to Taylor, Rezai, and Foley (2016)[44], this "paradox of thrift" characterizes profit-led adjustments in high-income countries. One of the issues at stake in this paper is to investigate to what extent climate damages may perturb such virtuous cyclical behaviour by preventing profitability from boosting investment.

As already said, when private debt accelerates too fast, it plays a pivotal role in destabilizing this desirable dynamic.

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[24] It is worth mentioning that both this "good" equilibrium and the "bad steady state" (with an unbounded debt ratio) turn out to be locally asymptotically stable, given our calibration and the analytical conditions found in section 3.2.

[25] This would induce an additional positive feedback on wages if workers did not share complete monetary illusion (i.e., $\gamma > 0$). Our empirical estimation at the world level – available in the supplementary web material – leads however to the value $\gamma = 0$, somewhat simplifying the dynamics.

Figure 2 shows that, over the course of this cyclical behavior, the world economy first tends to accumulate a growing debt ratio to finance investment (up to 160%) before slowly deleveraging. At the turn of this century, it remains at around 144%, while average real GDP growth is a quite reasonable 2.81%. So how does climate change modify this picture?

Let us begin by taking an analytical look at the stability properties of long-term equilibria.

3.2 Long-term equilibria with no inflation

The model presented in Section 2 boils down to a 16-dimensional nonlinear dynamical system. The economic and climate modules are coupled through: (i) the emissions defined in Eq. 14 and (ii) the damages specified in Eqs. 27, 30, 29, and 33. However, once the energy shift is fully completed, there are no longer any additional emissions and the climate module will converge to a unique stable equilibrium characterized by a constant positive mean atmospheric temperature deviation T_{eq}.[26] The economic and climate modules then become independent. As a result, an analysis of the macroeconomic module at its long-run climate equilibrium (if any) can be performed.

The system of differential equations characterizing the macroeconomic module reduces to

$$\begin{cases} \dot{\omega} & = \omega\left[\phi(\lambda) - (1-\gamma)i(\omega) - \alpha + \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{1-\mathbf{D}^{\mathrm{Y}}}\right] \\ \dot{\lambda} & = \lambda\left[g - \alpha - \beta + \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{1-\mathbf{D}^{\mathrm{Y}}}\right] \\ \dot{d} & = d\left[r - (g + i(\omega))\right] + \kappa(\pi) + \Delta(\pi) - (1-\omega), \end{cases} \tag{40}$$

with, as auxiliary variables, the growth rate of the population, of labor productivity, and of real output:

$$\begin{aligned} \beta &= q\left(1 - \frac{N}{P^N}\right), \\ \alpha &:= \frac{\dot{a}}{a} = g_a(T_{eq}), \\ g &:= \frac{\dot{Y}}{Y} = \frac{1}{\nu}(\kappa(\pi) - \mu G)(1 - \mathbf{D}^{\mathrm{Y}}) - (\delta + \mathbf{D}^{\mathrm{K}}) - \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{1-\mathbf{D}^{\mathrm{Y}}}, \\ &= \alpha + \beta + \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{1-\mathbf{D}^{\mathrm{Y}}}. \end{aligned}$$

Notice that, when damages to output vanish, the real growth rate becomes $g = \alpha + \beta$, as in a standard Solow or Ramsey growth model.

When climate has reached its long-run stationary state, environmental damages become constant, such that we can consider $G = 0$.[27] The complexity added by inflation makes it impossible to derive explicitly the long-run equilibria of the macroeconomic module. However, as shown by Grasselli and Nguyen-Huu (2015)[20] in a simpler framework, while the price dynamics of course alters the behavior of the system, it does not qualitatively change the phase space: in general, the dynamical system still admits two locally, asymptotically stable long-run equilibria. One of these has a non-zero employment rate and wage share, as well as a finite level of debt-to-output ratio. Let us call it a "good" equilibrium. The second long-run economic steady-state is characterized, on the contrary, by a zero employment rate and wage share, and an unbounded debt-to-output ratio – which we call a "bad " equilibrium.

For the sake of simplicity, in the rest of this subsection we assume that there is no inflation ($i(\omega) = 0$). Of course, we will relax this restriction for the numerical analysis in subsection 3.3, performed with the full-blown calibrated model. Within these restrictions, the system 40 admits a "good" equilibrium, whose growth rate is $g = \alpha$, provided the latter remains non-negative.[28] At this long-run steady state, the profit rate is defined by

$$\pi_{eq} = \kappa^{-1}\left(\nu\frac{\left(\alpha + \delta + \mathbf{D^{K}}_{eq}\right)}{1 - \mathbf{D^{Y}}_{eq}}\right),$$

while the remaining parameters of the equilibrium are

$$\begin{cases} \omega_{eq} & = 1 - \pi_{eq} - r d_{eq}, \\ \lambda_{eq} & = \phi^{-1}(\alpha), \\ d_{eq} & = \frac{\kappa(\pi_{eq}) + \Delta(\pi_{eq}) - \pi_{eq}}{\alpha}, \\ N_{eq} & = P^N. \end{cases} \tag{41}$$

That is, the long-run employment rate is a monotonic function of the steady-state real growth rate – which is a direct consequence of the short-term Phillips curve introduced in Eq. 12. Hence, if labor productivity declines due to global warming, so will the real growth and employment rates. As a result, the debt ratio will rise, thus increasing the repayment burden on the economy.

Furthermore, the combined effects of the increased profit rate and debt ratio will penalize the wage share, as they mechanically reduce the amount of remaining undistributed income. More precisely, at the "good" equilibrium, ω_1 can be rewritten as:

$$\omega_{eq} = 1 - \left(1 - \frac{r}{\alpha}\right)\pi_{eq} - r\frac{\kappa(\pi_{eq}) + \Delta(\pi_{eq})}{\alpha}. \tag{42}$$

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[26] For the sake of precision, the only remaining exogenous term of the climate module is F_{exo}. This exogenous forcing follows a sigmoid path defined by Eq. 24, and reaches its upper limit at equilibrium.

[27] Due to the price of the backstop technology in Eq. 35, abatement costs exponentially converge toward zero and can be considered null once the energy shift is completed.

[28] Given the quadratic specification for $\alpha = g_a(T_{eq})$ in Eq. 33, global warming may drive labor productivity growth into the negative domain. However, a negative growth rate of output would obviously lead the entire economy towards a collapse. Thus, $\alpha \geq 0$ is a *sine qua non* condition for the existence of a "good" equilibrium.

Therefore, a sufficient condition for the wage share to be penalized in the long run by global warming is that the equilibrium growth rate, α, be greater than the interest rate, r.[29]

3.3 A bifurcation induced by climate change

If we turn to the local asymptotic stability analysis of the long-run "good" equilibrium, its Jacobian matrix reads

$$M(\omega_{eq}, \lambda_{eq}, d_{eq}, N_{eq}) = \begin{bmatrix} 0 & M_{12} & 0 & 0 \\ -M_{21} & 0 & -rM_{21} & M_{24} \\ M_{31} & 0 & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{bmatrix},$$

where the entries $M_{ij}, (i,j) \in [\![1;3]\!]$ are given by:

$$\begin{aligned} M_{12} &:= \omega\phi'(\lambda) > 0, \\ M_{21} &:= \frac{\lambda_{eq}}{\nu}\kappa'(\pi_{eq})\big(1 - \mathbf{D}^{\mathrm{Y}}{}_{eq}\big) > 0, \\ M_{24} &:= \lambda_{eq}\frac{q}{P^N}, \\ M_{31} &:= \left(\frac{d_{eq}}{\nu}(1 - \mathbf{D}^{\mathrm{Y}}{}_{eq}) - 1\right)\kappa'(\pi_{eq}) - \Delta'(\pi_{eq}) + 1, \\ M_{33} &:= rM_{31} - \alpha, \\ M_{44} &:= -q. \end{aligned}$$

The characteristic polynomial $\chi_M(\cdot)$ of the Jacobian matrix at the "good" equilibrium writes

$$\begin{aligned} \chi_M(\epsilon) &= (\epsilon + q)\big[\epsilon^3 + (\alpha - rM_{31})\epsilon^2 + ... \\ &\quad ... + M_{12}M_{21}\epsilon + g_{Y_{eq}}M_{12}M_{21}\big]. \end{aligned}$$

The first root, $\epsilon = -q$, of the polynomial $\chi_M(\cdot)$ being obviously negative, the stability of the "good" equilibrium is given by the sign of the root of its factored polynomial of degree 3.[30] According to the Routh-Hurwitz criterion, a necessary and sufficient condition for the root of this polynomial to have a negative and non-null real part is

1. $\alpha > rM_{31}$,
2. $(\alpha - rM_{31})M_{12}M_{21} > \alpha M_{12}M_{21}$, which is equivalent to $rM_{31} < 0$ since M_{12} and M_{21} are positive, and α non-negative.

As a result, this necessary and sufficient condition boils down to $rM_{31} < 0$, that is:

$$r\left[\left(\frac{d_{eq}}{\nu}(1 - \mathbf{D}^{\mathrm{Y}}{}_{eq}) - 1\right)\kappa'(\pi_{eq}) + \Big(1 - \Delta'(\pi_{eq})\Big)\right] < 0.$$

This condition can be numerically verified within our calibrated model in order to infer the impact of climate change on the necessary and sufficient condition of stability for the “‘good” equilibrium.

Assuming $r > 0$ and $1 - \Delta'(\pi_{eq}) > 0$ (which will turn out to be the case for our empirical estimation), and using the equilibrium definition of the debt ratio, a necessary condition of stability can be provided with

$$\pi_{eq} > \frac{\nu(\delta + \mathbf{D}^{\mathrm{K}}{}_{eq})}{1 - \mathbf{D}^{\mathrm{Y}}{}_{eq}} + \Delta(\pi_{eq}).$$

As the equilibrium mean atmospheric temperature deviation increases, this expression makes explicit the destabilizing impact of global warming as the lower bound of the equilibrium profit rate rises, while this rate of course remains upper-bounded by 1. By "destabilizing impact," we mean that, on the one hand, the scope of the parameters for which the "good" equilibrium remains stable will reduce as global warming becomes more severe. On the other hand, within a given set of parameters, the global stability of the system weakens with climate change, as illustrated by the analysis of the long-run equilibrium with the level of temperature deviation provided in subsection 3.4.1.

3.4 Numerical analysis

We now turn to a numerical analysis of the impact of climate change on the long-run macroeconomic steady state of our economy. By contrast with the previous section, this time we take due account of the role played by inflation. Furthermore, the outcomes of our simulations now heavily depend upon our calibration of the dynamics, which is detailed in Appendix A.

As shown by Grasselli and Nguyen-Huu (2015)[20], in addition to adding more realism to the economic framework, inflation has a dampening effect on the endogenous real business cycles of the Keen model.

On the other hand, we saw previously that climate damages tend to reduce the wage share to some extent. Since, in this paper, inflation is cost-push, i.e., driven by the wage share, global warming will lower inflation and therefore increase the level of the debt-to-GDP ratio. Climate change will thus offset the dampening effect of inflation, again making destabilizing oscillations more plausible. Can we observe this effect at work? We address this question in the next two subsections.

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[29] Of course, this remark will have to be re-examined with an endogenous interest rate, set, for instance, by the central bank, as a function of inflation. We leave this for further research.

[30] The latter is similar to the characteristic polynomial found by Grasselli et al. (2012)[22].

3.4.1 The “good” steady state as a function of temperature

Echoing the discussion in subsection 3.2, let us consider in the phase diagram (ω, λ, d) how the long-run “good” equilibrium is affected by global warming. That is, in this subsection, we treat the temperature anomaly as an exogenous parameter (independent of abatement costs and emissions) and plot the corresponding long-run “good” equilibrium. To do so, we use the world calibration detailed in Appendix A and B, to simulate the system 40 together with an exogenously given temperature deviation.[31] We then numerically locate the long-run equilibrium. For this purpose, we consider environmental damages *à la* Weitzman allocated to output and capital (hereafter the Weitzman scenario introduced in Section 4). As shown by Figure 3, the steady state follows a quasi-linear trajectory (parameterized by temperature).

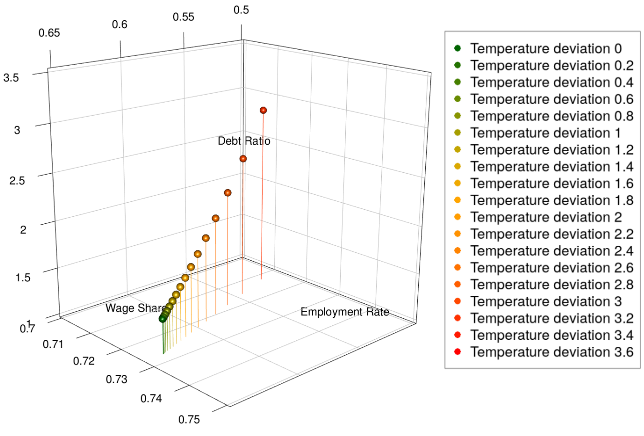

Figure 3: Trajectory of the steady state parameterized by temperature (Weitzman-type damages to output and capital).

In line with the mathematical analysis performed in Subsection 3.2, as the temperature anomaly progressively rises, the long-run private debt ratio increases while the wage share declines. As a result, the debt burden fuels a growing financial instability that ultimately prevents the economy from reaching the “good” equilibrium. The economy thus ends up in a “bad” equilibrium when the damages induced by global warming become too high (zero wage share and employment rate, unbounded debt ratio). It is worth mentioning that this bifurcation occurs with a temperature deviation close to +4°C. According to Lenton *et al.* (2008)[27], the +4°C threshold might indeed be a tipping point for the climate system. Our simulations suggest that, at this tipping point, the long-run employment rate should be close to 70%, the equilibrium wage share should shrink to 50%, and private debt should reach 300% of world GDP.

Notice that, in Figure 3, the growth rate of labor productivity was assumed to be exogenous. Consequently, the slight fall of the employment rate equilibrium value whenever global temperature increases is due to the decrease of inflation triggered by a diminishing wage share. This dynamical landscape changes substantially when we consider endogenous labor productivity growth as defined in Eq. 33, coupled with environmental Weitzman-type damages to capital (hereafter the Burke scenario introduced in Section 4), as shown in Figure 4.

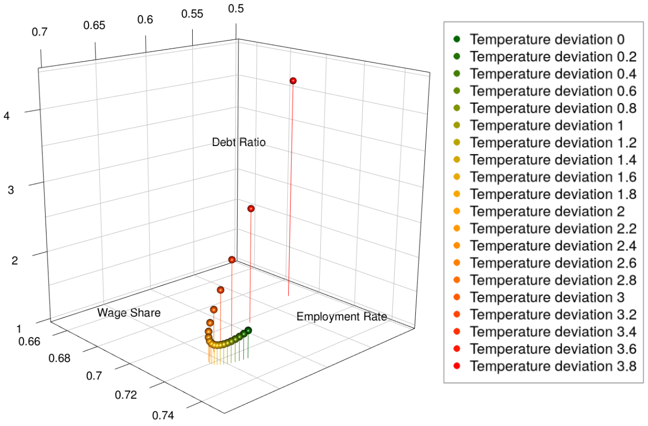

Figure 4: Trajectory of the steady state (Weitzman-type damages to capital and endogenous labor productivity).

The impact of endogenous labor productivity on the equilibrium growth rate, and the employment rate, is immediately visible: significant global warming induces a fall in the long-run real growth rate, and thus in employment. In addition, the relationship between global warming and labor productivity being nonlinear (in fact, quadratic), the impact of temperature is not a monotonic: with a low range of temperature deviations, warming increases the wage share and employment rate and decreases the private debt ratio. However, as soon as the temperature anomaly exceeds +2°C, the opposite impact can be observed – notably, a growing debt ratio that ultimately drives the economy into a severe breakdown.

3.4.2 A geometric view of the destabilizing effect of warming

As already said, climate change impacts the macrodynamics of the world economy inasmuch as it makes the “good” long-run equilibrium more difficult to reach. A bifurcation occurs whenever the “climate resistance” preventing the economy from converging towards this desirable steady state becomes too strong, such that the internal forces of the economy inevitably converge towards a collapse. Is it possible to have a clearer picture of how warming makes the “good” equilibrium less easy to reach? In fact, the change in the “good” equilibrium highlighted in Figures 3 and 4 is

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[31] Thus, the impact of global warming is quite simply captured by the influence of an exogenous temperature deviation on the damage function.

a long-term phenomenon that might not be noticeable in the short run. More specifically, it might be that the world economy is already following a path towards a "bad" steady state, without exhibiting much difference from the trajectories that would lead to the "good" steady state. Can we infer the long-run fate of our economy from short-run data?

We address these questions from a geometric perspective by comparing the basins of attraction of the "good" steady states without and with climate change. Varying initial conditions will possibly lead to various emission paths,[32] hence to different equilibrium temperature deviations and eventually to different "good" equilibria. Given some specified emission-reduction rate path, we therefore consider the set of *all* initial conditions that do *not* lead to an economic collapse. Let us call this set the "good" basin of attraction.

In order to numerically approximate the "good" basin of attraction, we adopted the following methodology. We started with a reasonable range of initial conditions for the variables of interest (wage share, employment rate, debt ratio), outside of which the world economy is definitely not viable.[33] We then considered another compact set to which long-term solutions must belong in order to be considered as economically desirable (hereafter the convergence set).[34] Any long-run steady state outside of this convergence set could hardly pretend to be a "good" equilibrium. Finally, we assumed a common emission-reduction rate path for the world economy defined here as the minimal path avoiding a collapse, given the postulated initial conditions. More precisely, we considered an initial real carbon price in 2010 of 2005 US$ 1 t/CO_2-e_2, in line with the calibration of the backstop technology path, and the associated minimal exponential growth rate necessary in order to avoid a collapse under the initial conditions presented in Appendix B. We then computed the trajectory starting anywhere in the initial set and checked whether they ended up in the convergence set at a large time scale. Whenever this was the case, the starting point was then considered to be part of the "good" basin of attraction.

We carried out this thought experiment assuming alternatively (i) no climate change (zero emissions) or (ii) temperature-dependent labor productivity coupled with environmental Stern-type damages to capital (hereafter the Stern scenario introduced in Section 4). Figure 5 first plots the "good" basin of attraction obtained with no global warming.

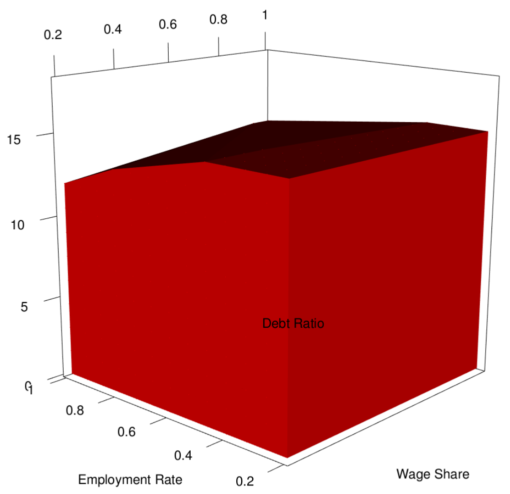

Figure 5: "Good" basin of attraction without climate change.

It turned out that almost all initial conditions led to the convergence set. This highlights the robustness of our model with respect to initial conditions: absent climate change, our modeling approach seems to promote an optimistic narrative in which the world economy converges to some rather desirable long-run steady state almost independently of its starting point. Figure 6 displays the "good" basin of attraction obtained in the Burke Extreme scenario.[35]

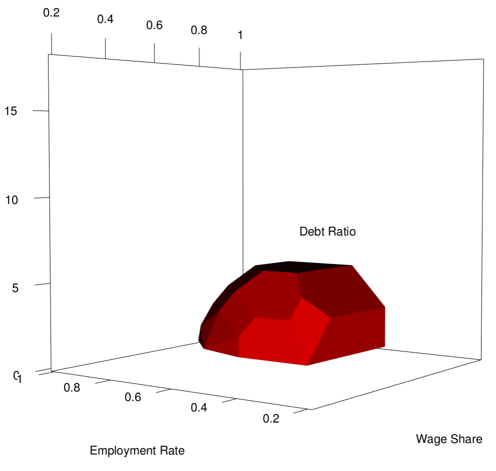

Figure 6: "Good" basin of attraction with Stern-type damages to capital and endogenous labor productivity.

Global warming obviously tends to narrow the set of initial conditions that would allow our world economy to

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[32] From this subsection on, and by contrast with subsection 3.4.2., the whole apparatus of climate feedbacks that links the world economy to our climate module is taken into account, so that the final temperature at the long-run steady state does depend upon the path folllowed by the economy.

[33] Our initial set is: $(\omega, \lambda, d) \in [0.2 : 0.99]^2 \times [0.1 : 18]$.

[34] The convergence set is: $(\omega, \lambda, d) \in [0.2 : 0.99]^2 \times [0.1 : 5]$.

[35] The computation of the basins of attraction for other damage types and allocation leads to similar results, namely, that as damages become more severe, the set of acceptable initial conditions under which a collapse may be avoided shrinks dramatically.

avoid an economic collapse. Labor productivity losses coupled with severe environmental damages to capital – both induced by global warming – require that the initial state of the world's trajectory be much closer to the $(1, 1, 0)$ point of our referential than when climate change is absent. This means that the higher the wage share today, the higher the employment rate and the lower the debt-to-output ratio, the easier it will be to circumvent a disaster. It is worth mentioning that the initial level of debt plays a prominent role, as an initially over-indebted economy would be incapable of carrying the additional burden of new debt resulting from investment triggered by climate change.

We are now ready to plunge into the details of the prospective scenarios with global warming.

4 Prospective analysis

This section aims to analyze the various prospective narratives that emerge from the set of scenarios we can envisage. It also suggests some public policy goals in order to achieve the objectives of limiting global warming to +2°C and ensuring financial stability. We close this section by discussing the feasibility of the more demanding objective adopted by the Paris Agreement to limit global warming as far as possible to +1.5°C by the end of this century.

4.1 The five scenarios

Our macroeconomic module was calibrated at the world level using data from the World Bank, Penn University, the Bureau of Economic Analysis, and the United Nations,[36] while the climate module was calibrated on the Nordhaus DICE model through a method of moments. Appendix A reports the details of this calibration, and Appendix B presents the initial values of our integrated dynamics. The path of our world economy is simulated over the period 2010–2300 in order to account for the long-term inertial effects of climate on the economy.

We consider five classes of scenarios. First, the *Baseline scenario* has been analysed in depth in subsection 3.1. Second, the *Nordhaus scenario* introduces Nordhaus-type climate damages and allows for a comparative analysis with the DICE model. Third, and in line with the recent literature (Dietz and Stern (2015) [10], Lenton *et al.* (2008) [27]), the *Weitzman scenario* deepens what the real impact of climate change might be, using a damage function *à la* Weitzman, allocated to both output and capital. Fourth, the *Burke scenario* takes advantage of Burke *et al.* (2015) [5] by introducing an alternative approach to modeling environmental damage. It thus combines an endogenous temperature-dependent labor productivity with Weitzman-type climate damages allocated to capital alone. Fifth, the *Stern scenario* explores a dramatic version of the previous scenario with Stern-type environmental damage allocated to capital. Figure 2 wraps up our five classes of scenarios.[37]

Scenario	**Baseline**	**Nordhaus**	**Weitzman**	**Burke**	**Stern**
Damage Type	-	Nordhaus	Weitzman	Weitzman	Stern
Damage on output	-	Yes	Yes	-	-
Damage on capital	-	-	Yes	Yes	Yes
Damage on labor productivity	-	-	-	Yes	Yes

Table 2: The five scenarios.

For each non-baseline scenario, we first examine (subsection 4.2.) the trajectory obtained together with a rather mild emission-reduction rate path, close to that considered by Nordhaus (2013) [35]. This policy is based on a real carbon price fixed in 2010 at an initial value of 2005 US$ 1 t/CO_2-e_2, together with an average growth rate of 2% per annum. We then study (subsection 4.3.) the display of more intensive emission-reduction paths that would make it possible to meet the +2°C temperature target —an objective that turns out to be sufficient to avoid a collapse.

4.2 Results with a baseline mitigation policy *à la* Nordhaus

Table 3 allows us to draw a first comparison between our five scenarios.

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[36] More precisely, the behavioral aggregate functions (i.e., the Phillips curve, investment, and dividends) were empirically estimated, while the remaining parameters were calibrated. See also footnote 5. Further details about our methodology are given in the supplementary web material.

[37] Of course, a number of other scenarios are conceivable – e.g., by coupling an endogenous temperature-driven productivity with damages *à la* Nordhaus, etc. They are available from the authors upon request.

Scenario	Baseline	Nordhaus	Weitzman	Burke	Stern
Average real GDP growth wrt 2010-2100	2.81%	2.75%	2.62%	1.73%	1.15%
Private debt ratio in 2100	1.44	1.65	3.19	1.73	7.50
CO_2 emissions per capita in 2050	-	7.82 t CO_2	7.77 t CO_2	6.41 t CO_2	5.26 t CO_2
Temperature change in 2100	-	+3.98°C	+3.96°C	+3.52°C	+3.49°C
CO_2 concentration in 2100	-	[illegible]	[illegible]	[illegible]	[illegible]

Table 3: Key value

At first, one observes that, as expected, global warming systematically penalizes output, since in our four last scenarios the average real growth rate between 2010 and 2100 remains below the Baseline scenario level of 2.81%. Moreover, climate change increases potential financial instability as shown by the higher private debt ratio reached at the end of this century. Next, a clear-cut distinction emerges between the exogenous and endogenous specifications of labor productivity. In fact, a temperature-dependent labor productivity (Burke and Stern scenarios) induces a sharper slowdown of output as the temperature rises, hence, lower CO_2 emissions (CO_2 concentration reaches 740 ppm in 2100 versus 960–975 ppm in the exogenous cases, i.e., in the Nordhaus and Weitzman scenarios). This results in a lower temperature deviation in 2100 (around +3.5°C compared to approximately +4°C in the exogenous cases) When damages affect capital (the Weitzman and Burke scenarios), temperature-dependent labor productivity seems to make the economy financially more resilient, since in 2100 the private debt ratio is a mere 173% in the Burke scenario compared to 319% in the Weitzman narrative. However, as we shall now see, the Burke scenario leads to a "secular stagnation" ultimately followed by collapse by the end of the twenty-second century. This suggests that the static picture of the world economy in 2100 captures very little of the whole story of climate and debt interaction. To capture this interaction, let us examine each scenario separately.

4.2.1 The Nordhaus scenario

We first turn to the Nordhaus scenario together with a carbon price trajectory (also) *à la* Nordhaus. Table 3 provides some key figures and Figure 8 the trajectories followed by our main macro variables within that scenario. Despite climate damages, productivity growth successfully drives exponential economic growth – real world GDP being multiplied by 10.5 between 2010 and 2100. As can be seen in Figure 7, the state of the world economy first fluctuates towards a long-run equilibrium relatively similar to that identified by the Baseline scenario.

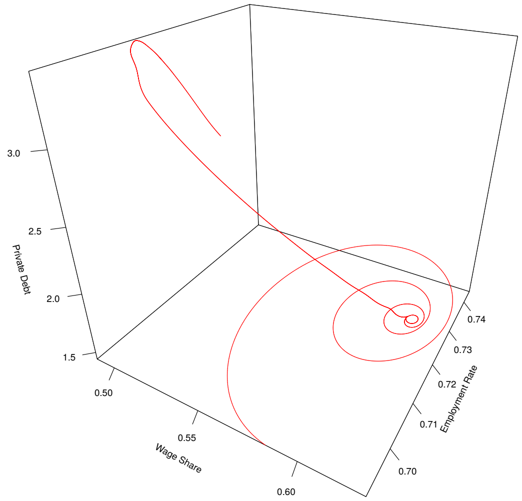

Figure 7: Phase diagram (ω, λ, d) for the Nordhaus scenario (period 2010–2600).

Yet, around the turn of the next century, due to high CO_2 emissions (up to approximately 147 Gt CO_2 in 2100), temperature increases (+3.98°C in 2100 in the atmospheric layer) substantially augment damages to production. These losses induce a significant rise in the debt ratio, driving the world economy far from its long-run stationary point. This illustrates the logic highlighted in Section 3. More precisely, the increase in environmental damages, $\dot{\mathbf{D}}^{\mathrm{Y}}$, is at its highest until the energy shift is completed.[38] At the same time, as the world population is plateauing, demography no longer contributes to output growth, which is driven solely by α, the labor-augmenting technological progress rate. Output, however, being penalized by climate change, remains below its long-run potential: $g < \alpha$. As a result, the employment rate, λ, further declines. Indeed, $L = Y/(a(1-\mathbf{D}^{\mathrm{Y}}))$, so that

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{L}}{L} = g + \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{(1-\mathbf{D}^{\mathrm{Y}})} - \alpha.$$

Since

$$g = \frac{(\kappa(\pi) - \mu G)(1-\mathbf{D}^{\mathrm{Y}})}{\nu} - \delta - \frac{\dot{\mathbf{D}}^{\mathrm{Y}}}{(1-\mathbf{D}^{\mathrm{Y}})},$$

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[38] Remember that, in the Nordhaus scenario, environmental damages only affect output.

we obtain

$$\frac{\dot{L}}{L} = \frac{(\kappa(\pi) - \mu G)(1 - \mathbf{D}^{\mathrm{Y}})}{\nu} - \delta - \alpha < 0 \quad \text{or close to } 0.$$

Meanwhile, the wage share, ω, decreases as a consequence of three forces conspiring together:

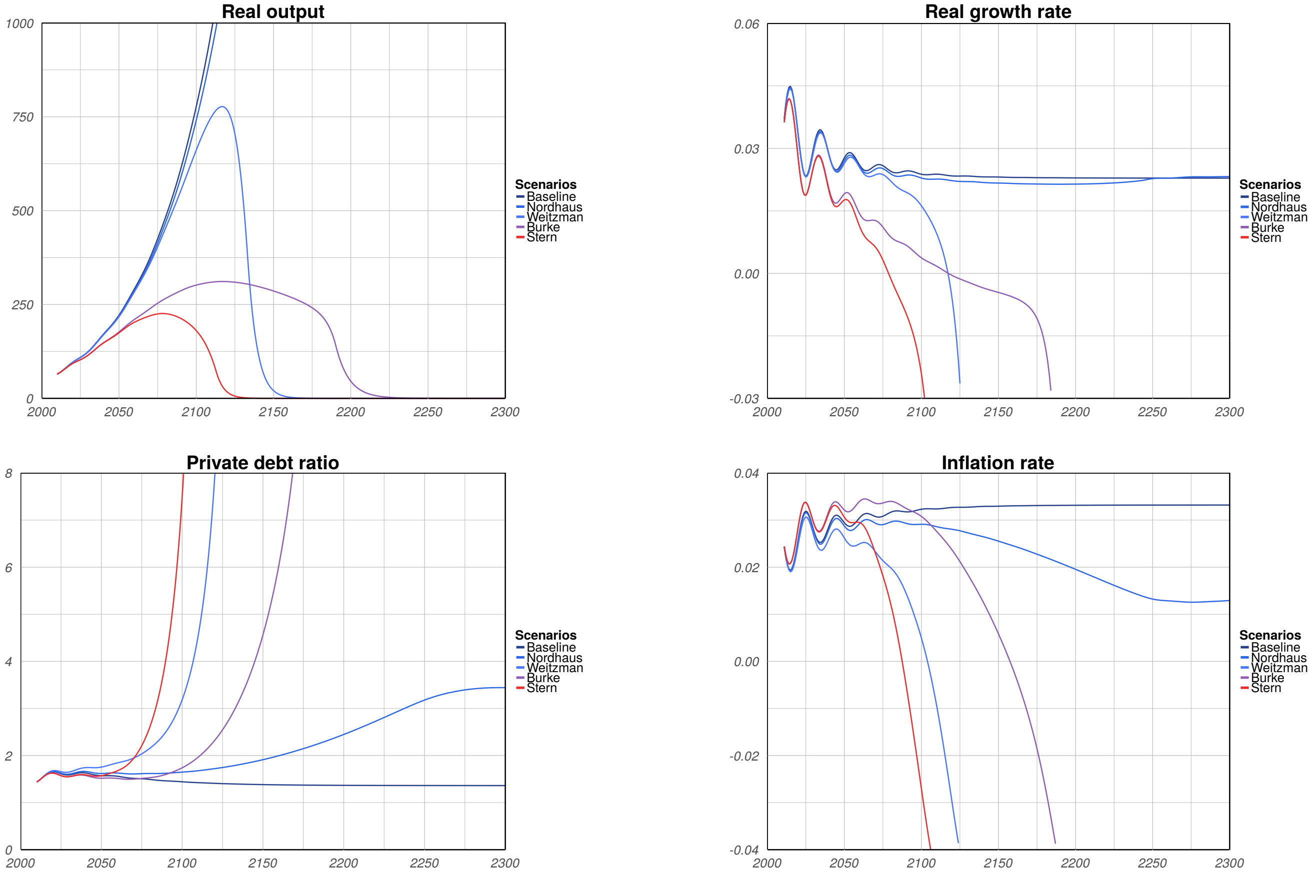
$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} + \frac{\dot{\lambda}}{\lambda} - i - g.$$

The decline in λ (with respect to its long-run stationary value) in fact reduces wages via the short-run Phillips curve, and $\dot{\lambda}/\lambda < 0$, while $i, g > 0$. As soon as damages decrease – courtesy of the (very slow) atmospheric cooling (after debt peaks) starting in 2250 as the energy shift is completed –, employment rises again and money wages once more begin to grow. Hence the (slow) return of the world economy to its long-run steady state. In other words, within this specific scenario, market forces are strong enough to counter the centrifugal forces of climate change, so that the (initial) long-run steady state will nevertheless be reached at a large time scale.

This scenario is quite reassuring: although the temperature deviation is far above the goal unanimously adopted at the Paris Agreement in 2015, the world economy seems to be going rather the well. The level of environmental damages remains below 20% of the world output while the temperature deviation reaches nearly $+9$°C in 2250.[39] As a result, CO_2 emissions peak only in the middle of the twenty-second century and the zero-emission level is reached one century later. This narrative confirms the rather unrealistic feature of the climate-economy interaction modeling on which it is based. As we shall now see, the picture changes dramatically as soon as damages are allocated between output and capital, or whenever endogenous labor productivity is considered.

4.2.2 When climate change becomes more severe: the Weitzman, Burke, and Stern scenarios

Figure 8 presents the deterministic trajectories of our model for the five scenarios under consideration (including the Baseline narrative), and Table 3 provides some related key figures.



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[39] Actually, Nordhaus' damage function is calibrated for a range of temperature anomalies between 0 and $+3$°C, and is not designed to capture catastrophic damages that might arise above $+3$°C (see Nordhaus (2013)[35]).

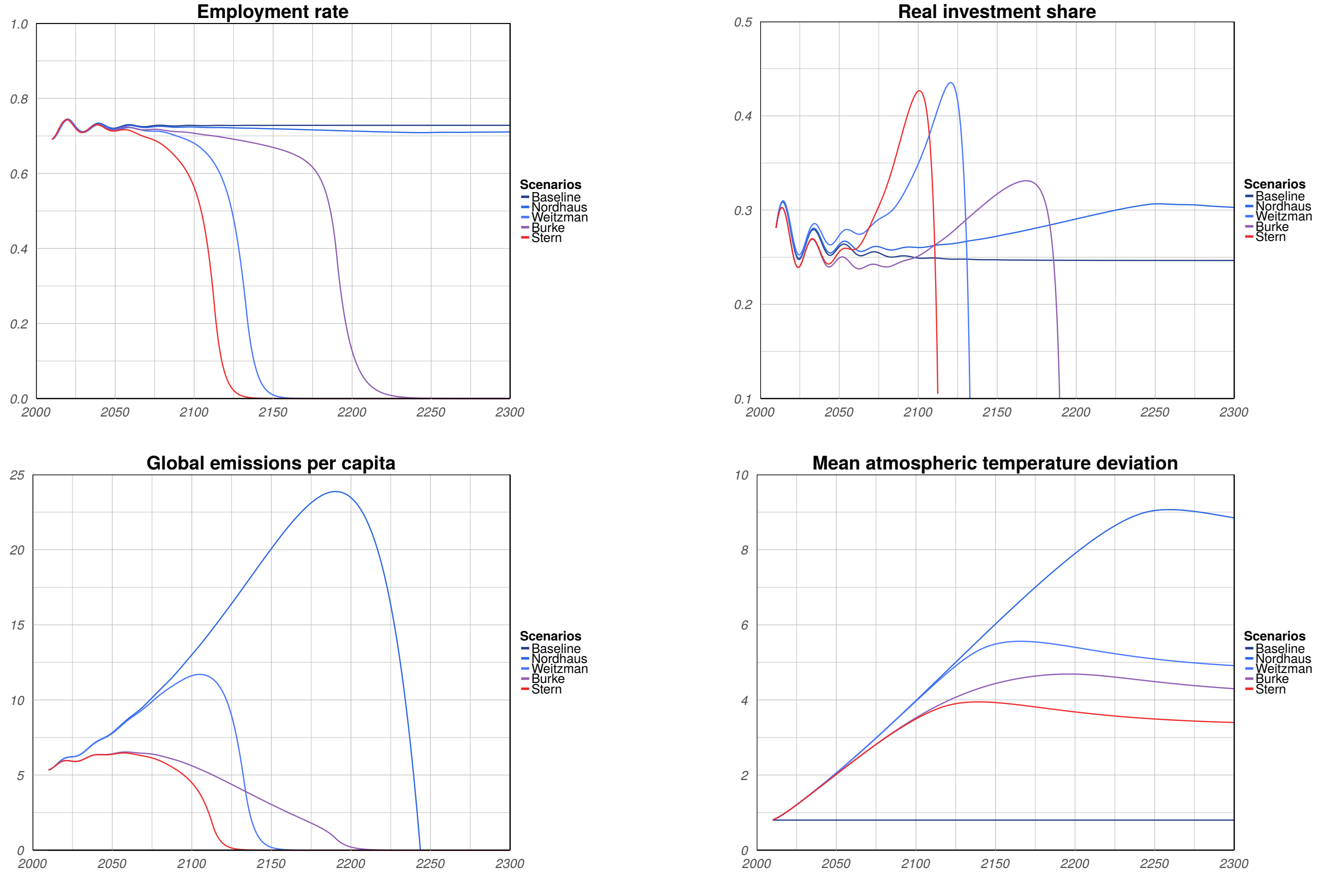

Figure 8: Trajectories of the main simulation variables in the proposed scenarios without proactive public policies.

All but the Baseline and Nordhaus scenarios exhibit an economic collapse of real output before the end of the twenty-second century, while the employment rate tends to zero and the debt ratio increases exponentially. Remember that Section 3 has shown that, with climate damages modeled in a somewhat more realistic fashion than through the Nordhaus damage function, global warming significantly shrinks the "good" basin of attraction. Our simulations therefore demonstrate that, at least within the conditions embodied in our last three scenarios, today's world economy lies outside the "good" basin of attraction shrunk by global warming. As a consequence, the "good" long-run steady state becomes out of reach.[40]

In the Weitzman, Burke, and Stern scenarios, climate change now inflicts direct damages on capital, permanently reducing potential output. Moreover, in the Burke and Stern scenarios labor productivity is endogenous and temperature-dependent. In fact, technological progress is steeply reduced by global warming as the temperature anomaly in the atmospheric layer exceeds 4°C, which is a reversal threshold for labor productivity growth.

As a result, the world productive sector is forced to leverage in order to finance investment, leading to a rise of the debt ratio, which heightens the risk of financial instability given the additional burden of debt service. Moreover, real output is penalized, ultimately generating disruptive effects on the labor market, while "forced" degrowth occurs. It is worth mentioning that the type of degrowth (by disaster, not by design) just alluded to is characteristic of Fisherian debt-deflation. Indeed, as discussed in Section 3, global warming penalizes the employment rate and the wage share, as observed in Fig. 8, while debt skyrockets.

Notice also that all these scenarios are accompanied by a temperature deviation in 2100 far above the +2°C target of the Paris agreement (in fact, higher than +3°C). Would a proactive emission-reduction rate path that manages to meet the +2°C imperative enable the world economy to avoid a collapse? And can we find some (possibly minimal) trajectory for a carbon price that would provide the right incentives to speed up the energy shift and ultimately achieve the +2°C target?

4.3 Achieving the +2°C target

The +2°C target has been evoked by the IPCC since 2003. First proposed by the European Union, this objec-

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[40] It is worth mentioning that a distribution of Nordhaus-type damages between output and capital (instead of just output as in the Nordhaus scenario) would also lead to a collapse by the end of the twenty-second century. Details are available from the authors upon request.

tive has been discussed in Bali (2007) before being mentioned in Copenhagen (2009) and finally adopted in Cancun (2010), and reaffirmed in Paris (2015). Given our exogenous backstop technology, a sufficiently fast-growing carbon price trajectory would *a priori* make it easier to implement a more intensive emission-reduction rate path, since it would then be more costly to continue emitting greenhouse gases. To avoid technicalities involved in variational calculus, we limit our analysis to the family of exponential carbon price paths.[41] In this setting, the mitigation trajectories can be characterized by two parameters: the initial carbon price and its (constant) annual growth rate.[42] Finally, we impose as a terminal condition a maximal temperature deviation of +2°C in 2100 in line with the public policy goal adopted by the Paris Agreement (2015). Figure 9 plots the results obtained in the set-up associated with the Weitzman scenario. Within this setting, a carbon price of, e.g., 2005 US$ 2.68 t/CO_2-e_2 in 2015,[43] dropping to $ 19.14 in 2025 and $ 136.93 in 2035 would be compatible with the achievement of the +2°C objective.[44]

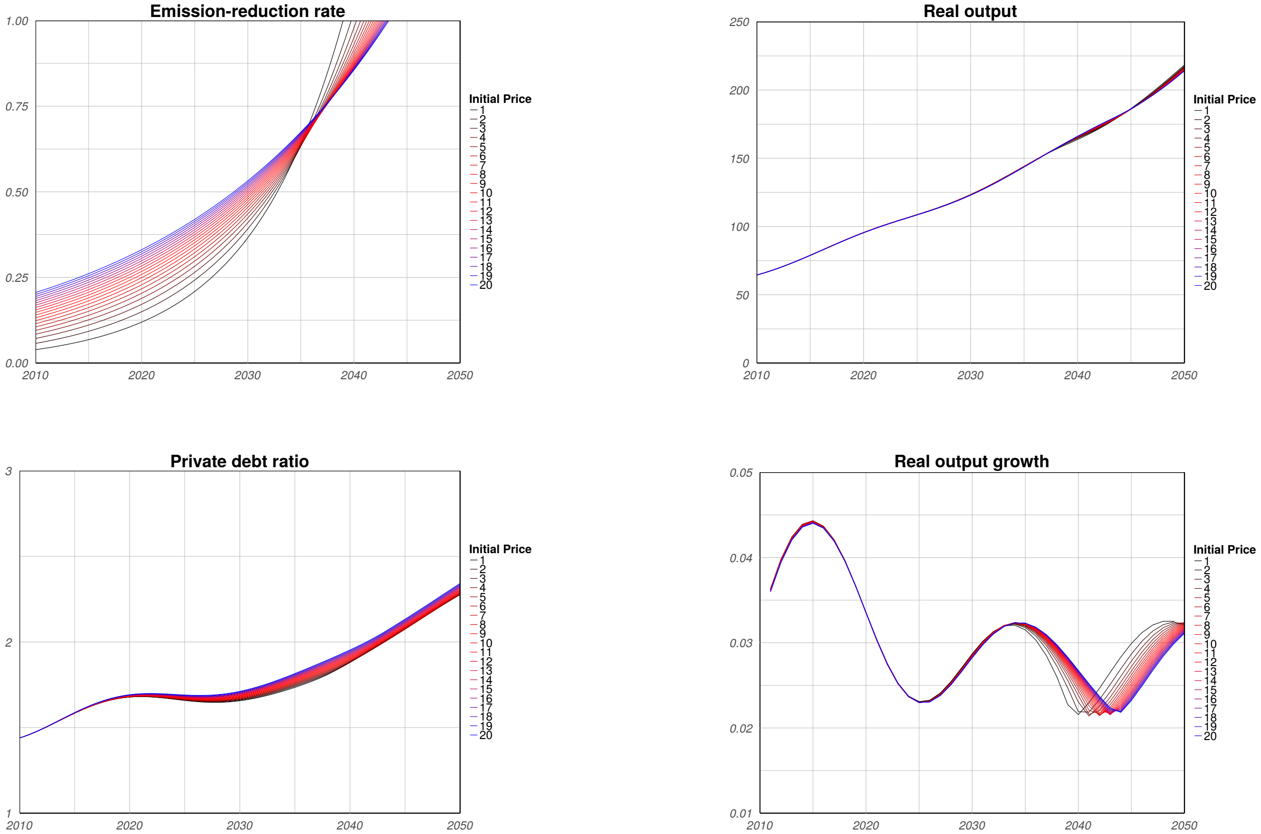

Figure 9: Weitzman scenario - trajectories of some key economic variables obtained with minimal carbon price paths.

All these trajectories exhibit a mean atmospheric temperature deviation of +2°C in 2100, courtesy of an energy shift performed around 2040. This means that, absent the implementation of technologies designed to capture the carbon stored in the atmosphere (i.e., negative net emissions), zero-emission needs to be reached at the world level as early as 2040 if the +2°C target is to be met. Moreover, the trajectories of the world real output are quite similar whatever the initial price, which is consistent with the fact that the average temperature in 2100 (and thus damages) is identical.[45] This is good news as it implies that there is no trade-off in terms of GDP growth: whatever emission-reduction rate path is chosen, if it is compatible with the +2°C target, the world economy will inevitably grow at a speed lower

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[41] The derivation of more general carbon price paths is left for further research.

[42] As defined in 2.3.2, the emission-reduction rate defines the fraction of CO_2 emissions avoided by implementing proactive climate policies. It is set through an arbitrage between the carbon price instrument and a backstop technology deployment cost (see Eq. 36).

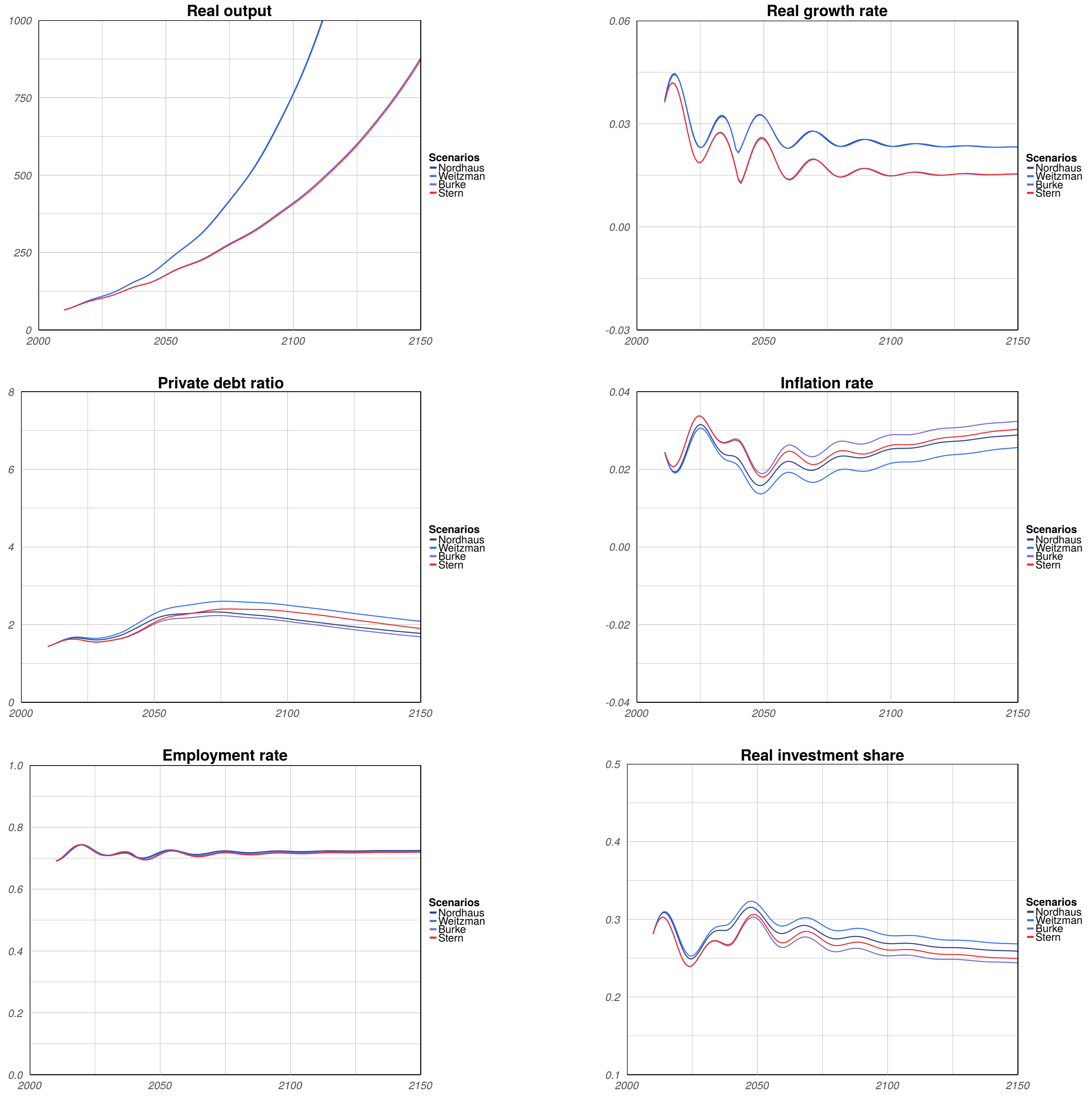
[43] That is, a price of 2005 US$ 2.68 for each unit of greenhouse-gas equivalent to a ton of CO_2.

[44] The same exercise carried out in the settings associated with the Burke and Stern scenarios leads to similar outcomes, at least up to the modelling year 2050. Indeed, the only way to meet the +2°C target turns out to consist in implementing the energy-shift early enough for the differences between the three above-mentioned scenarios to be almost negligible.

[45] The changes in abatement costs have little impact on the whole dynamics. Indeed, abatement costs always remain below 5% of real GDP.

than that identified in Figure 9, or in other words, around 2.75% per annum – at least as long as the zero net emission rate has not been reached. And yet, the choice of a mitigation path involves a number of policy options. Above all, the lower the initial carbon value, the higher its subsequent growth rate needed in order to meet the objective of limiting global warming, and the earlier the energy shift must be completed (the energy shift along the paths starting with the lower initial carbon value must be completed approximately ten years earlier than when the carbon price starts with the ligher initial value we have considered).

Having shed some light on the specific Weitzman setting, let us now consider our four classes of scenarios (distinct from the Baseline scenario) and compare, for each of them, the implementation of proactive public policies with minimal emission-reduction rate paths compatible with the +2°C temperature limitation objective. We use the methodology just described and select the minimal value for the exponential carbon price path so as to minimize the risk of financial instability with respect to the above-mentioned arbitrage. Figure 10 and Table 4 respectively present the deterministic trajectories and the key figures obtained for each of the scenarios.



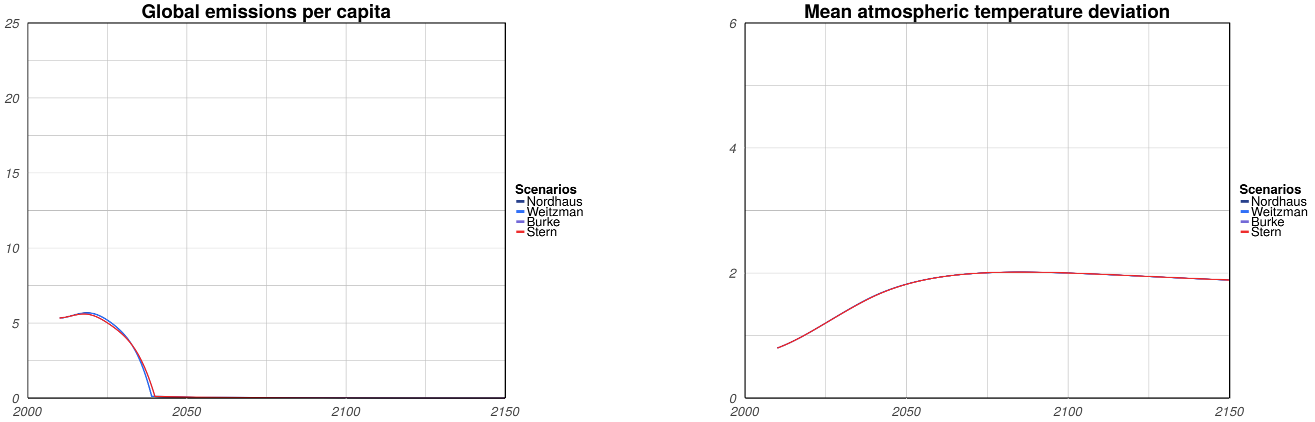


Figure 10: Trajectories of the main simulation variables in the proposed scenarios with a minimal carbon price path.

As expected, due to the similar emission-reduction rate path followed by the economy in each of them, the maximal temperature deviation reached in 2100 is also identical for all four scenarios (+2°C with an atmospheric CO_2 concentration of 396 ppm). From a macroeconomic perspective, all scenarios differ from the Baseline scenario in that they display a higher debt ratio (around 200% in 2100) and inflation rate (around 3% in 2100). Again, a sharp distinction emerges between scenarios induced by an exogenous labor productivity (Nordhaus and Weitzman), which remain close to the Baseline scenario in terms of real average output growth, and other scenarios (Burke and Stern), which are more distant from the baseline (with an average annual real output growth around 2% instead of 2.81%).

Scenario	**Baseline**	**Nordhaus**	**Weitzman**	**Burke**	**Stern**
Average real GDP growth wrt 2010-2100	2.81%	2.79%	2.78%	2.08%	2.07%
Private debt ratio in 2100	1.44	2.15	2.50	2.08	2.34
CO_2 emissions per capita in 2050	-	0.07 t CO_2	0.07 t CO_2	0.07 t CO_2	0.07 t CO_2
Temperature change in 2100	-	+2.00°C	+2.00°C	+2.00°C	+2.00°C
CO_2 concentration 2100	-	396 ppm	396 ppm	396 ppm	396 ppm

Table 4: Key values of the world economy.

4.4 The +1.5°C objective

We end our inquiry by examining the feasibility of the objective of limiting global warming to +1.5°C, also mentioned as being desirable by the Paris Agreement (art. 2).

Before describing our results, let us say one word about climate sensitivity, that is, the long-run temperature deviation that should result from a doubling of the pre-industrial atmospheric CO_2 concentration. So far, all our simulations of the climate module have been conducted under the assumption that climate sensitivity be equal to +2.9°C — which is the first mode of the sensitivity's probability distribution developed by Annan and Hargreaves (2006) [3].[46] Under this assumption, and given our modeling choices, it turns out that *there is no realistic carbon price path compatible with the realization of the* +1.5°C *objective*. Indeed, as illustrated by Figure 11 obtained under the Nordhaus scenario (the least severe scenario in terms of climate damages),[47] to attain this objective, the energy shift (hence zero net emission) would have to have been completed by 2016, which is obviously not the case. What is at stake here is the strong thermal inertia of the climate system, which drives global atmospheric temperature away from its current level even if no additional CO_2 emissions are released.

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[46] This parameter, however, is considered by most climate scientists as lying possibly in the $[1, 6]$ interval. To give the reader a flavour of the impact of this parameter, let us simply mention that, with a high climate sensitivity of 6°C, there is no way to fulfill the +2°C goal, at least within the Weitzman setting (i.e., with the weakest damage function). Indeed, even if we were to cut all emissions by 2015, the planet would reach a +2.29°C temperature anomaly by 2100, if $S = 6$°C.

[47] As with the analysis developed in 4.3, considering the other classes of scenarios would lead to similar results.

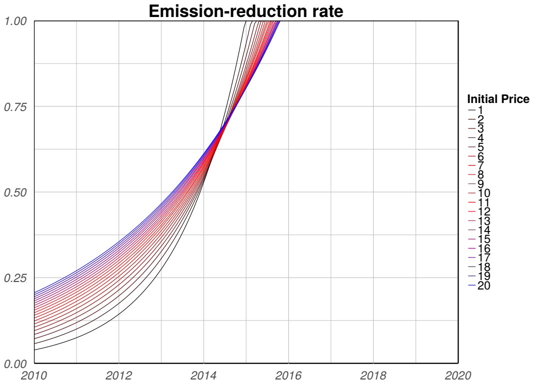

Figure 11: Minimal emission-reduction rate paths for the Nordhaus scenario under the +1.5°C objective.

In other words, with a +2.9°C climate sensitivity, it is too late to achieve the +1.5°C target. But what if we are lucky enough to have a +1.5°C climate sensitivity? We assume an initial carbon price between 1 and 20 to echo the discussion held in 4.3. Table 5 presents some illustrative results of the carbon price paths that would be required in order to reach the +1.5°C objective under a +1.5°C climate sensitivity. In order to emphasize the impact of an endogenous labor productivity, we restrict ourselves to the Weitzman and Burke scenarios .[48]

Scenario	**Weitzman**		**Burke**	
Carbon price 2015 (2005 US $)	1.68	24.65	1.63	24.31
Carbon price 2050 (2005 US $)	63.46	106.50	50.20	95.42
Annual growth rate of C price	10.38%	4.18%	9.79%	3.90%
Zero net emission	2063	2070	2067	2074

Table 5: Carbon price paths (in t/CO_2-e_2) compatible with the +1.5°C objective under a 1.5°C climate sensitivity.

Both scenarios require the energy shift to be completed by the end of 2075 through a rapid growth rate of the carbon price. A higher initial value (2005 US$20 t/$CO_2$-$e_2$) for the carbon price logically allows for a longer horizon to complete the energy shift (2070 instead of 2065). These results call for strong public policy intervention to drive the implementation of the energy shift. Indeed, even in the very optimistic case of a climate sensitivity of +1.5°C, the time horizon by which zero net emission needs to be achieved is rather short and requires a deliberately steep growth of the carbon price.

5 Conclusion

By combining financial and environmental aspects, the stock-flow consistent macroeconomic model introduced in this paper allows us to evaluate economic growth, or possible (forced) degrowth, depending on the dynamics of labor productivity, damages induced by global warming, climate sensitivity, as well as a carbon price path. To our knowledge, this is the first dynamic model estimated at the world level that enables both environmental and financial risks to be assessed within a framework of endogenous monetary business cycles.

Our main findings are as follows. When a relatively realistic growth path for technological progress is adopted, taking due account of the influence of global warming on labor productivity, a reasonable (i.e., significantly convex) damage function leads to a possible breakdown of planetary magnitude either short before or around the next century's turning point.

Second, our simulations shed new light on the interplay between financial (level of private debt) and climate instabilities. In line with the previous work of Dafermos (2016)[8], our simulations suggest both financial and climate instabilities reinforce each other and may ultimately lead to a planetary economic collapse. Besides, increasing the wage share, fostering employment and reducing the private debt-to-output ratio would make it easier for today's world economy to belong to the "good" basin of attraction, i.e., to be able to find a path circumventing a breakdown. In other words, coping with collapse on a hotter planet means, among other things, private deleveraging, income distribution in favor of workers, and a high employment rate. To the best of our knowledge, it is the first time that these channels are shown to facilitate adaptation to climate change.

Third, the implementation of an adequate policy of emission-reduction through the deployment of a carbon price trajectory enables long-term prosperity to be restored whenever climate sensitivity is 2.9°C. According to the simulations performed in this paper, however, the binding carbon price trajectory must be such that – either starting at a high level or rapidly increasing – the energy shift be completed and zero net emission be reached as early as 2040 and, in any case, *before 2050*. For instance, within our

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[48] As already shown in our previous analysis, our scenarios can be distinguished according to whether they assume labor productivity to be exogenous or endogenous.

framework, a carbon price of, e.g., 2005 US$ 3 t/CO_2-e_2 in 2015, $ 20 in 2025 and $ 137 in 2035 is compatible with the achievement of the +2°C objective. At Paris, nearly 200 countries promised to try to bring global emissions down from peak levels as soon as possible. More significantly, they pledged "to achieve a balance between anthropogenic emissions by sources and removals by sinks of greenhouse gases in the second half of this century."

Experts say that this means getting to "net zero emissions" between 2050 and 2100. The UN's climate science panel says net zero emissions must happen by 2070 to avoid dangerous warming.

On the other hand, it seems too late for the world economy to be able to prevent the average temperature from exceeding the +1.5°C threshold, unless with a stroke of luck climate sensitivity turns out to be very low (+1.5°C or less). Given the radical uncertainty that plagues climatologists' knowledge about climate sensitivity, these results call for strong and immediate action.

These results *a posteriori* justify our choice not to follow a standard cost-benefit analysis to assess the impact of climate-driven externalities. Certainly, the latter approach inevitably ends up with the issue of calibrating the "right" discount rate. While substantial efforts have been devoted to assessing whether a high or low, and sometimes a time-varying, discount rate should be considered,[49] none of this literature, to the best of our knowledge, has ever considered a negative rate.[50] Yet, this possibility should be seriously envisaged. Not only because of the pervasive negative interest rates observed nowadays on international markets, but also, as shown in this paper, because a world breakdown might be the prospect that markets should start facing from now on. If the next generation is going to be less wealthy than we are today, then a US dollar today should be worth less than the same dollar in a couple of decades.

Finally, this paper calls for a number of extensions. Let us only mention a few of them for the sake of brevity: can an appropriate tax policy implement the type of income redistribution that would favor adaptation to climate change? It would be good to allow for some substitutability between labor and capital in the world production function. Would it ease the task of the production sector when compensating losses inflicted by global warming? Or would it favor unemployment, and hence, following the findings of this paper, make it harder to circumvent a collapse? What if, instead of behaving myopically, economic actors were to share (possibly wrong) expectations about the near future? And finally, would curbing the demographic curve – say, by means of some systematically implemented family planning policy – enable us to reach the 1.5°C challenge more easily? All of this raises crucial challenges for future research.

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[49] see, e.g., Sterner and Persson (2008)[42]

[50] Except for Ekeland (2015)([12], p.49), who introduces an "ecological interest" rate. According to Ekeland, consumption goods (available in large quantities) and natural resources (available in limited quantities) should be valued using two different interest rates. While the first one can be set by the market, the second one should be lower or negative due to its finiteness.

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Appendices

A Calibration of the Model

Symbol	Description	Value	Remarks and sources
c	The constant mean inflation	0.03	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
C	The heat capacity of the atmosphere, biosphere and upper ocean	50.76142	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
C_0	The heat capacity of deeper ocean	16.52632	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
$CO2_{AT}$	The CO$_2$ pre-industrial concentration in the atmosphere layer	588 Gt C	The DICE model, Nordhaus (2013)[35]
$CO2_{UP}$	The CO$_2$ pre-industrial concentration in the biosphere and upper ocean layer	1 350 Gt C	The DICE model, Nordhaus (2013)[35]
$CO2_{LO}$	The CO$_2$ pre-industrial concentration in the deeper ocean layer	10 000 Gt C	The DICE model, Nordhaus (2013)[35]
E	The initial level of industrial CO$_2$ emission of the economy	33.61 Gt CO2	The DICE model, Nordhaus (2013)[35]
$F_{2\times \mathrm{CO}_2}$	The change radiative forcing resulting from a doubling of CO$_2$ concentration wrt to pre-industrial period	3.8 W/m^2	The DICE model, Nordhaus (2013)[35]
F^P_{exo}	The upper bound of the exogenous radiative forcing	0.7 W/m^2	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
f_K	The fraction of environmental damage allocated to the stock of capital	1/3	Dietz and Stern (2015)[10] and Moyer *et al.* (2014)[30]
m	The mark-up of the price dynamics	1.60997	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
P^N	The upper limit of the population dynamics	$7.05592.10^9$	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
q	The speed of growth of the population dynamics	0.02744	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
r	The interest short-term rate of the economy	$7.05592.10^9$	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
S	The climate sensitivity parameter	2.9 °C	The DICE model, Nordhaus (2013)[35]
T_{preind}	The preindustrial temperature	13.74°C	NASA (2016)[2]
Y	The initial real output level of the economy	$64.4565.10^9$	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
α	The constant growth rate of labor productivity	0.0226	Empirically estimated, macroeconomic database, more details available in the web supplementary material
α_1	Parameter of the endogenous labor productivity growth	0.0071 /°C	Burke *et al.* (2015)[5]
α_2	Parameter of the endogenous labor productivity growth	- 0.0004 /°C^2	Burke *et al.* (2015)[5]
$\gamma*$	The degree of monetary illusion of the economy	0	Empirically estimated, macroeconomic database, more details available in the web supplementary material
δ	The depreciation rate of capital	0.06253	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
Δ_0	The constant value of the $\Delta(.)$ function	- 0.1395	Empirically estimated, macroeconomic database, more details available in the web supplementary material
Δ_1	The slope value of the $\Delta(.)$ function	0.9001	Empirically estimated, macroeconomic database, more details available in the web supplementary material
$\delta_{E_{Land}}$	The growth rate of land use change CO$_2$ emissions	- 0.04	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
$\delta_{F_{exo}}$	The convergence speed of the exogenous forcing	0.25	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
δ_{g_σ}	The variation rate of the growth of emission intensity	- 0.001	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
δ_{p_C}	Exogenous growth rate of the carbon price	-	Variable of scenario
$\delta_{p_{BS}}$	Exogenous growth rate of the back-stop technology price	- 0.001	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
ζ_3	Damage function parameter	6.754	The DICE model, Nordhaus (2013)[35]
η_p	The relaxation parameter of the inflation	.08197	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
θ_1	Parameter of the abatement cost function	0.0001	The DICE model, Nordhaus (2013)[35]
θ_2	Parameter of the abatement cost function	2.8	The DICE model, Nordhaus (2013)[35]
κ_0	The constant value of the $\kappa(.)$ function	0.04260	Empirically estimated, macroeconomic database, more details available in the web supplementary material
κ_1	The slope value of the $\kappa(.)$ function	0.64153	Empirically estimated, macroeconomic database, more details available in the web supplementary material
μ	The fraction of the abatement costs borne by investment	0	Variable of scenario
ν	The constant capital-to-output ratio	2.8956	Empirically calibrated, macroeconomic database, more details available in the web supplementary material
π_1	Damage function parameter	0 /°C	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
π_2	Damage function parameter	0.00284/°C^2	The DICE model, Nordhaus (2013)[35]
π_3	Damage function parameter in the Weitzman case	0.00000507/°C$^{\zeta_3}$	Dietz and Stern (2015)[10]
π_3	Damage function parameter in the Stern case	0.0000819/°C$^{\zeta_3}$	Dietz and Stern (2015)[10]
ϕ_0	The constant value of the $\phi(.)$ function	- 0.73502	Empirically estimated, macroeconomic database, more details available in the web supplementary material
ϕ_1	The slope value of the $\phi(.)$ function	1.08519	Empirically estimated, macroeconomic database, more details available in the web supplementary material
Φ_{12}	Transfer coefficient for carbon from the atmosphere to the upper ocean/biosphere	0.01727	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework
Φ_{23}	Transfer coefficient for carbon from the upper ocean/biosphere to the lower ocean	0.0005	The DICE model, Nordhaus (2013)[35], adjusted for a continuous framework

The mentioned macroeconomic database gathers data from the World Bank, Penn University, the Bureau of Economic Analysis and the United Nations.

B Initial values of the Model

Symbol	Description	Value	Remarks/sources
CO_2^{AT}	The CO_2 concentration in the atmosphere layer	830.4 Gt C	The DICE model, Nordhaus (2013)[35]
CO_2^{UP}	The CO_2 concentration in the biosphere and upper ocean layer	1527 Gt C	The DICE model, Nordhaus (2013)[35]
CO_2^{LO}	The CO_2 concentration in the deeper ocean layer	10 010 Gt C	The DICE model, Nordhaus (2013)[35]
d	The private debt ratio of the economy	1.43931	Empirically calibrated, macroeconomic database
E_{land}	The exogenous land use change CO_2 emissions	3.3 Gt CO_2	The DICE model, Nordhaus (2013)[35]
F_{exo}	The exogenous radiative forcing	0.25 W/m^2	The DICE model, Nordhaus (2013)[35]
g_σ	The growth rate of the emission intensity of the economy	- 0.01	The DICE model, Nordhaus (2013)[35]
p	The composite good price level	1	Normalization constant
p_C	The carbon price level	1	Variable of scenario
p_{BS}	The back-stop price level	344	The DICE model, Nordhaus (2013)[35]
N	The workforce of the economy	$4.55100.10^9$	Empirically calibrated, macroeconomic database
T	The temperature in the atmosphere, biosphere and upper ocean layer	0.8 $^\circ$C	The DICE model, Nordhaus (2013)[35]
T_0	The temperature in the deeper ocean layer	0.0068 $^\circ$C	The DICE model, Nordhaus (2013)[35]
λ	The employment rate of the economy	0.691	Empirically calibrated, macroeconomic database
ω	The wage share of the economy	0.58496	Empirically calibrated, macroeconomic database

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